



UNIT 5

MEASUREMENT

OBJECTIVES

After completion of this unit, you should be able to solve mathematics problems involving linear, area, circular, and volume measurement. This knowledge will be evidenced by correctly completing the Assignment Sheets and by scoring a minimum of 70% on the Unit Test.

Specifically, you should be able to:

1. Solve linear measurement problems.
2. Solve area measurement problems.
3. Solve circular measurement problems.
4. Solve volume measurement problems.

Each of these objectives is covered in the pages that follow.

OBJECTIVE 1

LINEAR MEASUREMENT

Measurement is an important part of the BAC Craftworker's job (*Figure 5.1*). **Linear** means something that is in line or straight. A **linear measure** involves measuring straight-line distances between two points.



Figure 5.1 BAC Craftworker Measuring Linear Distance

TWO BASIC MEASURING SYSTEMS

Linear measurement in the BAC Craftworking trade is done using two basic systems. The one used most often in the United States is the "inch" system. The system used in Canada and most other countries is the metric, or SI, system. The *Advanced Mathematics* course provides more information on linear measurement using the metric system.

As seen in Unit 1, the inch can be divided into fractions of an inch (e.g., $\frac{1}{2}$ ", $\frac{3}{4}$ ", $\frac{1}{16}$ ", $\frac{2}{32}$ ", 0.38", 0.75", etc.). There are also multiples of an inch, including the foot and the yard. Of course, if you have enough inches, feet, and yards, you can end up with a mile. However, most measurements and conversions used most often performed by the Craftworker involve inches and feet. *Table 5.1* shows the basic linear measurements used by a Craftworker.

LINEAR MEASURE EQUIVALENTS		
1 foot	=	12 inches (12")
1 yard	=	3 feet (3')
1 yard	=	36 inches (36")

Table 5.1 Basic Linear Measurement Conversions

MEASURING AND CONVERTING

There will be many times when you will need to convert feet to inches and inches to feet. There may even be occasions when you need to do conversions involving yards. Complicating linear measurement and conversions is that some drawings will use mixed numbers to indicate a distance (e.g., 6' 9 ½") and other drawings may use decimal parts of a foot (e.g., 12.3').

To convert from a decimal part of a foot to inches, first multiply the decimal fraction times 12 inches. Suppose that a drawing indicates a distance of 14.8' and you want to know this distance in feet and inches. It takes a few steps to convert.

Example: Convert 14.8' to feet and inches as a mixed number with a common fraction.

We know that we have 14' and 0.8, or 80%, of another foot. This means that we have $0.8 \times 12" = 9.6"$.

$$9.6" = 9 \frac{6}{10}" = 9 \frac{3}{5}"$$

$$\text{So, } 14.8' = 14' 9 \frac{3}{5}"$$

Of course, you will not find $\frac{3}{5}"$ on most tapes and rules.

Note: To find a fraction that will work with the inch ruler, first change $\frac{3}{5}$ to a decimal. This is $3 \div 5 = 0.6$. Then, multiply 0.6 times the measurement or accuracy you wish to work with (e.g., sixteen, thirty-seconds). Assume you are working with sixteen, then $0.6 \times 16 = 9.6$, which is between $\frac{9}{16}$ and $\frac{10}{16}$ (or $\frac{5}{8}$). As 9.6 sixteenths is closer to 10 sixteenths or $\frac{5}{8}$, you would use $\frac{5}{8}$. This gives you an answer of 14'9 $\frac{5}{8}"$.

CONVERTING WITH A CALCULATOR

Construction calculators are designed to perform the conversions seen in the previous problem automatically. Using a construction calculator to convert 14.8' to feet and inches as a mixed number will look something like this:



Figure 5.2 Construction Calculator Performing Conversions

Figure 5.2 shows the calculator displays for the previous problem. Review the following conversion examples.

Example: Convert 6.75' to feet and inches as a mixed number with a common fraction.

$$0.75 \text{ is } 75\% \text{ of a foot, or } .75 \times 12" = 9"$$

Answer: 6.75' = 6' 9".

Example: Convert 15' 3" to feet and inches as a mixed number with a decimal fraction.

We know that we have 15' and then 3" out of the next 12". This is $\frac{3}{12}$ or 0.25, or 25%, of 1 foot.

Answer: 15' 3" = 15.25'.

Example: Convert 8' 3 $\frac{3}{16}$ " to feet and inches as a mixed number with a decimal fraction.

$$3 \frac{3}{16}" = 3.1875" \text{ (divide 16 into 3 to convert to a decimal)}$$

We have 3.1875" out of the next 12". This is $\frac{3.1875}{12} = 0.266$ (rounded to 3 decimal places).

Answer: 8' 3 $\frac{3}{16}$ " = 8.266'. (This is approximately the answer, as we rounded to 3 decimal places.)

Solving the previous problem using a construction calculator will look something like this:





Figure 5.3 Construction Calculator Performing Conversions

Figure 5.3 shows the calculator display for the previous problem.

CONVERTING FEET TO INCHES

Let's work the previous problem in reverse : Convert 8.266' to feet and inches using common fractions.

Example: $0.266' \times 12'' = 3.192''$

Assuming that we are working in sixteenths, then we have:

$0.192'' \times 16 = 3.072''$

Answer: $8.266' = 8' 33/16''$. (This is approximately the answer, as we rounded in the original problem.)

Parts of an Inch

In the BAC Craftworking trades, the basic tools used for linear measurement include the tape measure, ruler, or scale. As we learned in a previous unit, each inch is then divided into parts (or fractions). These include:

- Halves ($\frac{1}{2}$)
- Quarters ($\frac{1}{4}$)
- Eighths ($\frac{1}{8}$)
- Sixteenths ($\frac{1}{16}$)
- Thirty-seconds ($\frac{1}{32}$ – only on some rulers and tapes)
- Sixty-fourths ($\frac{1}{64}$ – only on some rulers and tapes)

Figure 5.4 is a drawing of part of a ruler or tape showing several inches. Note that the inches are divided into sixteenths (top scale) and eighths (bottom scale).

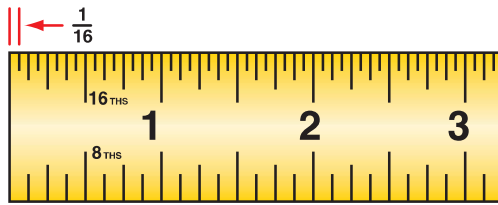


Figure 5.4 Portion of a Measuring Tape

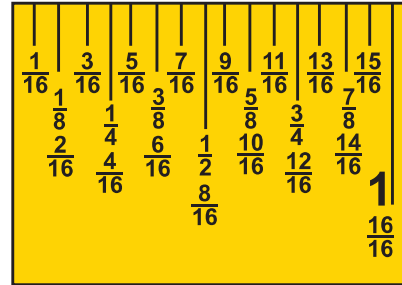


Figure 5.5 Fractional Parts of an Inch in Sixteenths

Figure 5.5 is a drawing of an inch showing the fractional parts down to sixteenths.

TAKING LINEAR MEASUREMENTS

When taking a linear measurement, follow these steps.

1. Place the beginning (or 0") of the ruler or tape at one end of the line you are measuring whenever possible. This will help prevent making an error in reading the length between the two points.
2. Place the ruler or tape next to the point at the other end of the line you are measuring.
3. Determine the distance from the beginning of the ruler or tape to the endpoint. First, determine the whole number of feet and/or inches. For example, the line ends somewhere after 7" and before 8". So, you know the linear measure is 7".
4. Look within that last inch (between 7" and 8"). The length is determined by locating the fraction mark closest to the point. For example, you determine that the line is next to $\frac{5}{16}$ ". So, the length of the line is $7\frac{5}{16}$ ".

The following lines are specific lengths. Use a tape or ruler to verify the length of each line.

$2\frac{1}{2}$ "



$3\frac{3}{4}$ "



$4\frac{7}{8}$ "



ADDING AND SUBTRACTING LINEAR MEASURES

Many times, on blueprints, you will need to add and subtract linear measurements. The process of adding and subtracting mixed numbers was covered in a previous unit. Here are some examples of how to add and subtract linear measurements involving mixed numbers.

Example: Add $4' 3 \frac{3}{4}'' + 7' 6 \frac{7}{16}''$.

It may be easier to see this problem in columns.

$$\begin{array}{r} 4' 3 \frac{3}{4}'' \\ + 7' 6 \frac{7}{16}'' \\ \hline \end{array}$$

Step 1: Begin by finding the LCD shared between the fractions $\frac{3}{4}$ and $\frac{7}{16}$. That number is 16. After multiplying the numerator (3) times 4 and the denominator (4) times 4, we get $\frac{12}{16}$.

$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$

$$\begin{array}{r} 4' 3 \frac{12}{16}'' \\ + 7' 6 \frac{7}{16}'' \\ \hline \end{array}$$

Step 2: Now that we have like denominators, let's add the two fractions. We are given $\frac{19}{16}''$ in the inches column.

$$\frac{12}{16} + \frac{7}{16} = \frac{19}{16}$$

$$\begin{array}{r} 4' 3 \frac{12}{16}'' \\ + 7' 6 \frac{7}{16}'' \\ \hline 19 \frac{19}{16}'' \end{array}$$

Step 3: Convert the improper fraction $\frac{19}{16}''$ into a mixed fraction. We have 19 sixteens. This is $1 \frac{3}{16}$. Since the 1 carries, we are left with $\frac{3}{16}''$.

$$\frac{19}{16} = 1 \frac{3}{16}$$

$$\begin{array}{r} 4' 3'' \\ 7' 6'' \\ + 1 \frac{3}{16}'' \\ \hline 3 \frac{3}{16}'' \end{array}$$

Step 4: Add the inches: 3, 6, and the 1" that we carried.

$$\begin{array}{r} 4' 3'' \\ 7' 6'' \\ + 1' 3/16'' \\ \hline 10' 3/16'' \end{array}$$

Step 5: Add the feet.

$$\begin{array}{r} 4' 3'' \\ 7' 6'' \\ + 1' 3/16'' \\ \hline 11' 10' 3/16'' \end{array}$$

Answer: 11' 10' 3/16"

SOLVING BY CALCULATOR

Solving the previous problem using a construction calculator will look something like this:

4 FEET 3 INCH 3 / 4 + 7 FEET
6 INCH 7 / 16 = 11 10-3/16

Figure 5.6 shows the calculator display for the previous problem.



Figure 5.6 Construction Calculator Performing Addition

Example: Subtract $8' 7 \frac{3}{8}" - 4' 8 \frac{11}{16}"$.

Step 1: Write the problem in columns.

$$\begin{array}{r} 8' 7 \frac{3}{8}" \\ - 4' 8 \frac{11}{16}" \\ \hline \end{array}$$

Step 2: Begin by finding the lowest common denominator (LCD) for the fractions $\frac{3}{8}"$ and $\frac{11}{16}"$. The LCD is 16, so convert $\frac{3}{8}"$ to sixteenths, which is $\frac{6}{16}"$.

$$\begin{array}{r} 8' 7 \frac{6}{16}" \\ - 4' 8 \frac{11}{16}" \\ \hline \end{array}$$

Step 3: As $\frac{6}{16}"$ is smaller than $\frac{11}{16}"$, we will need to borrow $\frac{16}{16}"$ (or 1") from the 7" (leaving 6"). This gives us $\frac{22}{16}" - \frac{11}{16}" = \frac{11}{16}"$.

$$\begin{array}{r} 8' 6 \frac{22}{16}" \\ - 4' 8 \frac{11}{16}" \\ \hline 11 \frac{11}{16}" \end{array}$$

Step 4: Because we borrowed 1" from 7", we have 6". As 6" is smaller than 8", we will need to borrow 12" (or 1') from the 8' (leaving 7'). This gives us $6" + 12" = 18"$. $18" - 8" = 10"$.

$$\begin{array}{r} 18 \\ 7' 6 \frac{22}{16}" \\ - 4' 8 \frac{11}{16}" \\ \hline 10 \frac{11}{16}" \end{array}$$

Step 5: Because we borrowed 1' from 8', we now have 7'. $7' - 4' = 3'$.

$$\begin{array}{r} 7' 18 \frac{22}{16}" \\ - 4' 8 \frac{11}{16}" \\ \hline 3' 10 \frac{11}{16}" \end{array}$$

Answer: $3' 10 \frac{11}{16}"$

CONVERTING MEASURES TO DECIMAL FRACTIONS

If you need to multiply and divide linear measures in mixed numbers, it is best to first convert the measurements to decimal fractions, and then multiply or divide.

Example: Multiply $2' 5 \frac{3}{8}" \times 4' 6 \frac{1}{2}"$.

Step 1: Convert $5 \frac{3}{8}"$ to a mixed number with a decimal fraction:

Divide 3 by 8, and add this answer to 5".

$$\begin{array}{r} \frac{3}{8} \\ .375 + 5 \\ 5.375" \end{array}$$

The equation becomes: $2' 5.375" \times 4' 6 \frac{1}{2}"$

Step 2: Convert inches to feet: Since we have 5.375" out of 12", divide $\frac{5.375}{12}$ and add this answer to 2'.

$$\begin{array}{r} 5.375 \div 12 = .45 \text{ (rounded to 2 decimal places).} \\ .45 + 2 = 2.45' \end{array}$$

The equation becomes $2.45' \times 4' 6 \frac{1}{2}"$

Step 3: Convert $4' 6 \frac{1}{2}"$ to a mixed number with a decimal fraction: Divide $\frac{1}{2}$ and add this answer to 6".

$$\begin{array}{r} \frac{1}{2} \\ .5 + 6 \\ 6.5" \end{array}$$

The equation becomes: $2.45' \times 4' 6.5"$

Step 4: Convert inches to feet: Because we have 6.5" out of 12, divide $\frac{6.5}{12}$, and add this answer to 4'.

$$\begin{array}{r} 6.5 \div 12 = .54 \text{ (rounded to 2 decimal places)} \\ .54 + 4 = 4.54' \end{array}$$

The equation becomes $2.45' \times 4.54' =$

Step 5: Multiply the decimal fractions.

$$2.45' \times 4.54' = 11.12 \text{ sq ft (square feet to 2 decimal places)}$$

Answer: $2.45' \times 4.54' = 11.12 \text{ sq ft}$

SOLVING BY CALCULATOR

You should be able to do this calculation with your scientific calculator.
The problem looks like this:

$$(2 + [5 + 3 \div 8] \div 12) (4 + [6 + 1 \div 2] \div 12) =$$

The sequence on your scientific calculator will be similar to the following:

(2 + (5 + 3 ÷ 8) ÷ 12) × (4 + (6 + 1 ÷ 2) ÷ 12) = 11.12

The display should read 11.12 (to 2 decimal places).

This problem would be much easier using a construction calculator. Here are the steps using a construction calculator:

Multiply $2' 5 \frac{3}{8}" \times 4' 6 \frac{1}{2}"$.

2 FEET 5 INCH 3 / 8 × 4 FEET 6 INCH 1 / 2 = 11.12

CONVERTING MIXED UNIT MEASUREMENTS

Consider another example of converting fractions to decimals. This problem shows feet and inches being divided by inches. To solve it, convert to either all feet or all inches, as shown in the next two examples.

Example: Divide $14' 9 \frac{11}{16}" \div 4 \frac{3}{4}"$ — converting all measures to inches.

Step 1: First, convert 14' to inches by multiplying it by 12".

$$14' =$$

$$14' \times 12" \text{ per foot} =$$

$$168"$$

Step 2: Convert $\frac{11}{16}"$ to a decimal by dividing 11 by 16 and adding the answer to 9.

$$\frac{11}{16}" = .6875 + 9 = 9.6875"$$

Step 3: Add the values converted to inches.

$$168'' + 9.6875'' = 177.6875''$$

Step 4: Convert $4\frac{3}{4}''$ into a decimal by dividing 3 by 4 and adding the answer to 4.

$$4\frac{3}{4}'' = 4.75''$$

Step 5: Divide $177.6875''$ by $4.75''$.

$$177.6875'' \div 4.75'' = 37.41 \text{ (rounded to 2 places)}$$

There is no unit of measurement in this answer, because the inches cancel out. This means that 4.75 will divide into 177.6875 approximately 37.41 times.

Answer: 37.41

Example: $14' 9 \frac{11}{16}'' \div 4 \frac{3}{4}''$ — converting all measures to feet.

Step 1: Convert $14' 9 \frac{11}{16}''$ into a decimal fraction.

$$14' 9 \frac{11}{16}'' =$$

$$14 + (9 + 11 \div 16) \div 12 =$$

$$14.81' \text{ (rounded to 2 places)}$$

Step 2: Convert $4 \frac{3}{4}''$, or $4.75''$, into feet by dividing by 12 (there are 12 inches in 1 foot, and we have 4.75 of those inches).

$$4.75'' \div 12'' \text{ per foot} = 0.4' \text{ (rounded to 2 places)}$$

$$14.81' \div 0.4' = 37.03 \text{ (rounded to 2 places).}$$

Answer: 37.03

USE CAUTION WHEN ROUNDING

In the previous problem, we determined the answers using two different solutions. The answers were 37.41 and 37.03. Why the difference? These answers should be the same. The reason for the difference is that in several places we rounded off the answer. By rounding off, we added some error to the problem. When you then multiply or divide, this error is often magnified. So, use caution when rounding numbers.

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 2

AREA MEASURE

Area, or **surface**, measure refers to measurement of something that has width and length (or width and height). The piece of plate in *Figure 5.7* has length, width, and a surface. The shape of this piece of plate is that of a **rectangle**.

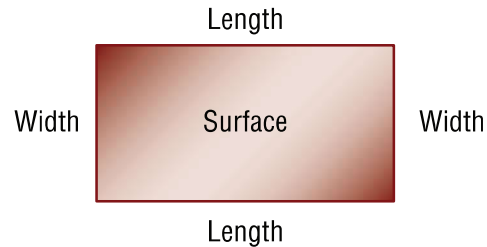


Figure 5.7 Rectangle

This rectangle has four sides. By definition, the two lengths are **parallel** and are the same. Likewise, the two widths are parallel and are the same.

Parallel means that the sides run side-by-side (similar to train tracks – they are a fixed distance apart). This also means that the four corners form 90° angles. *Figure 5.8* shows one of the four corners enlarged.

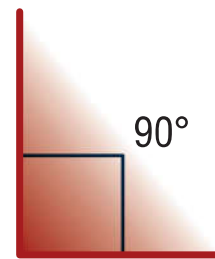


Figure 5.8 Right Angle

The **perimeter** of this rectangle is the distance around the four sides of the rectangle. So, if the length of a rectangle is 24" and width is 17", then the perimeter is $24" + 24" + 17" + 17" = 82"$.

The surface of this rectangle has length and width, but it does not have thickness. Although the plate does have thickness, we are presently talking only about surface measure.

DETERMINING UNITS OF MEASUREMENT

To determine the area, or size, of the surface, the first step is to make sure that both the length and width are in the same units of measurement (e.g., inches, feet, or meters). The area of a surface is expressed in **square units**. For example, if you are determining the area of the steel plate and the length and width are in inches, then the area is expressed in **square inches**. This is also written as **sq in**, or **in²**.

$$\text{Inches} \times \text{inches} = \text{square inches}$$

$$\text{Feet} \times \text{feet} = \text{square feet}$$

$$\text{Meters} \times \text{meters} = \text{square meters}$$

CALCULATING AREA OF A RECTANGLE

Here are the steps to determine the area of a rectangle.

Step 1: Ensure that the length and width are in the same units of measure.

Step 2: Multiply length times the width. Written as an equation, this looks like:

$$A = l \times w$$

$$\text{Area} = \text{length} \times \text{width}$$

Step 3: Express the product in lowest terms and in square units of measure.

Review the following examples.

Examples: Find the area of a panel 14" long and 12" wide.

$$\begin{aligned} A &= l \times w \\ &= 14'' \times 12'' \\ &= 168 \text{ square inches (or sq in)} \end{aligned}$$

Find the area of a panel of window glass 2'6" tall and 18" wide, first in square feet and then in square inches.

Because 2'6" = 2.5' and 18" = 1.5', we can calculate:

$$\begin{aligned} A &= l \times w \\ 2.5' \times 1.5' &= 3.75 \text{ sq ft} \end{aligned}$$

Because 2'6" = 30", we also can calculate:

$$\begin{aligned} A &= l \times w \\ 30'' \times 18'' &= 540 \text{ sq in} \end{aligned}$$

SQUARE UNITS BY CALCULATOR

Construction calculators are able to perform these calculations without first making any conversions. Enter this sequence to solve the previous problem using a construction calculator.

$$\boxed{2} \boxed{\text{FEET}} \boxed{6} \boxed{\text{INCH}} \boxed{\times} \boxed{1} \boxed{8} \boxed{\text{INCH}} \boxed{=} \\ \boxed{3.75} \boxed{\text{SQ FT}} \boxed{\text{INCH}} \boxed{=} \boxed{540} \boxed{\text{SQ IN}}$$

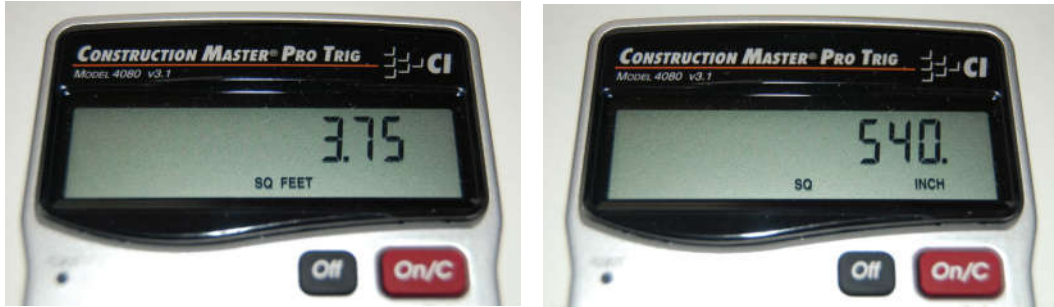


Figure 5.9 Calculating Area

Figure 5.9 shows the two answer displays from the previous problem on a construction calculator.

CALCULATING AREA OF A SQUARE

A **square** is a shape with all four sides being of equal length. A square is shown in Figure 5.10. Note that sides 1, 2, 3, and 4 are all the same length. All four angles within the square are 90° .

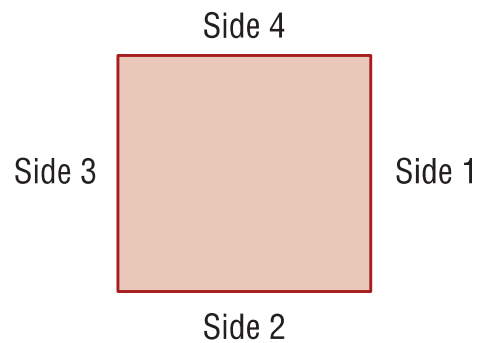


Figure 5.10 Square

The formula for calculating the area of a square is the same as that for a rectangle: $A = l \times w$. However, because length equals width, you can also write this formula as:

$$A = s^2$$

$$A = \text{length of 1 side squared}$$

Review the following examples of calculating the area of a square.

Examples: Find the area of a square room that measures 12 feet on one side.

$$A = s^2 = 12^2$$

$$12' \times 12' = 144 \text{ sq ft}$$

Find the area (in square feet) of a square piece of decking that measures 3' 9" on one side.

$$A = s^2 = (3' 9")^2$$

$$3.75' \times 3.75' = 14.0625 \text{ sq ft (14.06 sq ft to 2 places)}$$

Find the area of a piece of square plate that measures 1' 7 ½" on one side.

Convert 1' 7 ½" to a decimal and express in feet:

$$1 + 7.5 \div 12 = 1.625'$$

$$A = s^2$$

$$1.625' \times 1.625' = 2.64 \text{ sq ft (rounded to 2 places)}$$

CALCULATING AREA TO DETERMINE WEIGHT

On the work site, you may need to calculate area to determine the weight of various work materials. *Table 5.2* shows the weights per square foot of various types of metal.

WEIGHTS OF METALS PER SQUARE FOOT								
Thickness in inches	Weights, pounds per square foot							
	Wrought iron	Cast iron	Steel	Copper	Tin	Zinc	Brass	Lead
1/16	2.50	2.34	2.55	2.89	2.41	2.28	2.63	3.7
1/8	5.00	4.69	5.10	5.79	4.81	4.55	5.26	7.4
3/16	7.50	7.03	7.65	8.68	7.22	6.83	7.89	11.1
1/4	10.00	9.38	10.20	11.60	9.63	9.10	10.50	14.8
5/16	12.50	11.70	12.80	14.50	12.00	11.40	13.20	18.5
3/8	15.00	14.10	15.30	17.40	14.40	13.70	15.80	22.2
7/16	17.50	16.40	17.90	20.30	16.80	15.90	18.40	25.9
1/2	20.00	18.70	20.40	23.20	19.30	18.20	21.10	29.7
9/16	22.50	21.10	23.00	26.00	21.70	20.50	23.70	33.4
5/8	25.00	23.50	25.50	28.90	24.10	22.80	26.30	37.1
11/16	27.50	25.80	28.10	31.80	26.50	25.00	28.90	40.8
3/4	30.00	28.10	30.60	34.70	28.90	27.30	31.60	44.4
13/16	32.50	30.50	33.20	37.60	31.30	29.60	34.20	48.2
7/8	35.00	32.80	35.70	40.50	33.70	31.90	36.80	51.9
15/16	37.50	35.20	38.30	43.40	36.10	34.10	39.50	55.6
1	40.00	37.50	40.80	46.30	38.50	36.40	42.10	59.3

Table 5.2 Weights of Types of Metals

Example: Calculate the weight of a 6 foot by 4 foot piece of $\frac{1}{4}$ " steel plate.

Step 1: The first step is to calculate the area of the plate.

$$6' \times 4' = 24 \text{ sq ft}$$

Step 2: Consult *Table 5.2* to determine the weight of a square foot of steel. Each square foot of a $\frac{1}{4}$ " plate weighs 10.20 pounds.

Step 3: Calculate the total weight of the $\frac{1}{4}$ " piece of steel:

$$24 \text{ sq ft} \times 10.20 \text{ pounds/sq ft} = 244.8 \text{ pounds}$$

Answer: 244.8 pounds

INTRODUCTION TO TRIANGLES

In a square and rectangle, there are four sides and four angles inside the figure or shape. A **triangle** (*Figure 5.11*) has three sides and three interior angles. Triangles will be covered in more depth in *Advanced Mathematics*.

The triangle shown in *Figure 5.11* is a right triangle. This means the angle between the base of the triangle and height (or altitude) of the triangle is 90° .

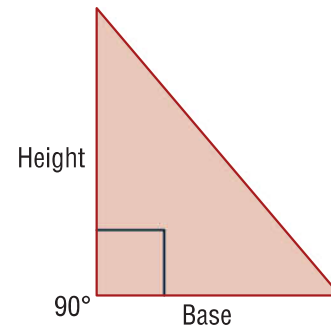


Figure 5.11 Triangle

A right triangle is one-half of a rectangle or square. Note in *Figure 5.12* that when a line is cut diagonally from one corner to the opposite corner, two right triangles are created.

The triangles shown are right triangles. There are several types of triangles. The triangle shown in *Figure 5.13* does not contain a **right angle**.

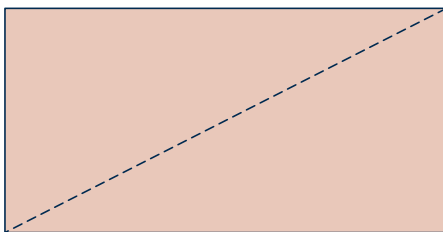


Figure 5.12 Rectangle is Two Right Triangles

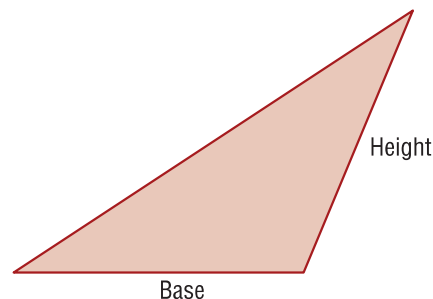


Figure 5.13 Triangle with No Right Angles

DETERMINING AREA OF A TRIANGLE

To determine the area of a triangle, multiply the base times one-half of the height. Here is the formula:

$$A = \frac{1}{2} bh \quad A = \frac{1}{2} \times \text{base} \times \text{height}$$

Use the formula to calculate triangle area in the following problems.

Examples: What is the area of a triangle that has a base of 7" and height of 11"?

$$A = \frac{1}{2} bh$$

$$0.5 \times 7" \times 11" = 38.5 \text{ sq in}$$

Find the area of a right triangle with a base of 6'10" and height of 9' 3 3/4". Calculate in square feet to 2 decimal places.

$$6'10"$$

$$6.83' \text{ (to 2 places)}$$

$$9' 3 \frac{3}{4}"$$

$$9.31' \text{ (to 2 places)}$$

$$A = \frac{1}{2} bh$$

$$0.5 \times 6.83' \times 9.31' = 31.79 \text{ sq ft}$$

CALCULATE TRIANGLE AREA BY SCIENTIFIC CALCULATOR

Let's solve the previous problem using a scientific calculator. Be sure that you understand why each keystroke is being made.

• 5 × (6 + 10 ÷ 12) ×
(9 + 3 . 75 ÷ 12) = 31.82

Note that when you compare the two answers (31.79 and 31.82), the difference is due to the rounding off in the first example.

CALCULATE AREA BY CONSTRUCTION CALCULATOR

Now solve this problem using a construction calculator.

$$\begin{array}{cccccccccccc} \cdot & 5 & \times & 6 & \text{FEET} & 10 & \text{INCH} & \times & 9 \\ \text{FEET} & 3 & \text{INCH} & 3 & / & 4 & = & \text{31.82} \end{array}$$

DETERMINING AREA WITH MULTIPLE SHAPES

Some problems will require several steps to solve, as they involve two or more shapes within one shape. Review the following examples.

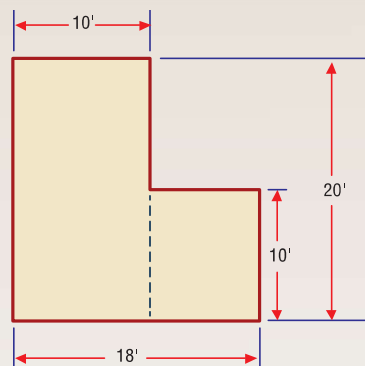


Figure 5.14 Area of Floor Section

Example: Determine the area of the section of floor shown in *Figure 5.14*.

Step 1: Determine the shapes of the area measured. Note that within this figure, we have 2 rectangles. If we draw a dotted line, it becomes clear that we have one rectangle that is $10' \times 20'$ and a second rectangle that is $10' \times 8'$.

Step 2: Determine the formula for calculating area of each shape and add subtotals to find the total area. The total area would be the sum of the two smaller areas.

$$A = l \times w + l \times w$$

Step 3: Plug the measurements for each shape into the formula to calculate total area.

$$\begin{aligned} 10' \times 20' + 10' \times 8' \\ 200 \text{ sq ft} + 80 \text{ sq ft} = 280 \text{ sq ft} \end{aligned}$$

Answer: 240 sq ft

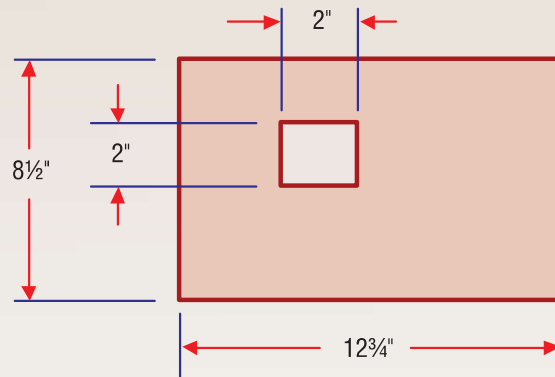


Figure 5.15 Area of Plate

Example: The piece of plate in *Figure 5.15* has a square opening cut in the plate. What is the area of the plate remaining after the square is cut out?

Step 1: Calculate the area of the entire plate.

$$A = l \times w$$

$$8.5" \times 12.75" = 108.375 \text{ sq in}$$

Step 2: Calculate the area of the square cut out.

$$A = s^2$$

$$2^2 = 4 \text{ sq in}$$

Step 3: Calculate the area of the plate after the cut out:

$$108.375 \text{ sq in} - 4 \text{ sq in} = 104.375 \text{ sq in}$$

Answer: 104.375 sq in

INTRODUCTION TO PARALLELOGRAMS

A **parallelogram** is a figure or shape within which the two lengths are parallel and the two heights (or widths) are parallel. However, the sides are not necessarily at right angles (90°) as they are for rectangles. A parallelogram is shown in *Figure 5.16*.

If we draw a dotted line on the right side of the parallelogram, beginning at the base and coming straight up until we reach the top, we have created a triangle. If we cut off the triangle and attach it to the other end (shown on the left side of *Figure 5.17*), then we transform the parallelogram into a rectangle.

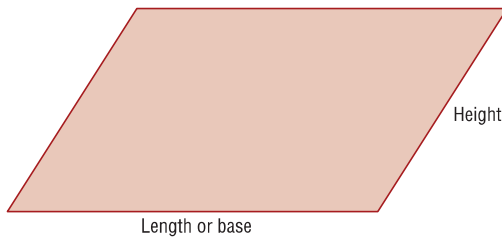


Figure 5.16 Parallelogram



Figure 5.17 Transforming a Parallelogram into a Rectangle

CALCULATING AREA OF A PARALLELOGRAM

Calculate the area of a parallelogram using the same formula as we did with the rectangle:

$$A = l \times w \quad \text{Area} = \text{length} \times \text{width}$$

CALCULATING AREA OF A TRAPEZOID

A **trapezoid** (*Figure 5.18*) is a four-sided figure or shape. In order to be a trapezoid, two of the sides (called *bases*) must be parallel.

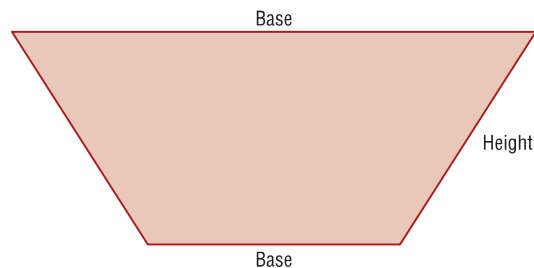


Figure 5.18 Trapezoid

Follow these steps to calculate the area.

1. Ensure that all units of measure are the same.
2. Add the lengths of the two bases.
3. Multiply the sum by one-half of the height.

Calculating the area can be expressed in the following formula.

$$A = (b_1 + b_2) \times 0.5h$$

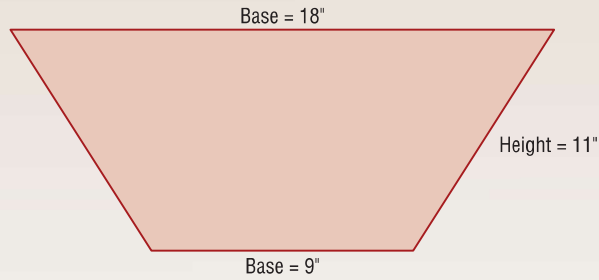


Figure 5.19 Calculating Area of a Trapezoid

Example: Calculate the area of the trapezoid shown in *Figure 5.19* according to the established formula:

$$A = (b_1 + b_2) \times 0.5h$$

The steps to calculate the area of the trapezoid in *Figure 5.19*:

$$\begin{aligned} A &= (9'' + 18'') \times 0.5 \times 11'' \\ &= 27'' \times 5.5'' \\ &= 148.5 \text{ sq in} \end{aligned}$$

COMPUTING TRAPEZOID AREA BY CALCULATOR

Let's solve the previous problem using a scientific calculator:



Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 3

CIRCULAR MEASURE

The tire on a forklift is **circular** in shape (*Figure 5.20*). When we look at the tire, we see a **circle**.

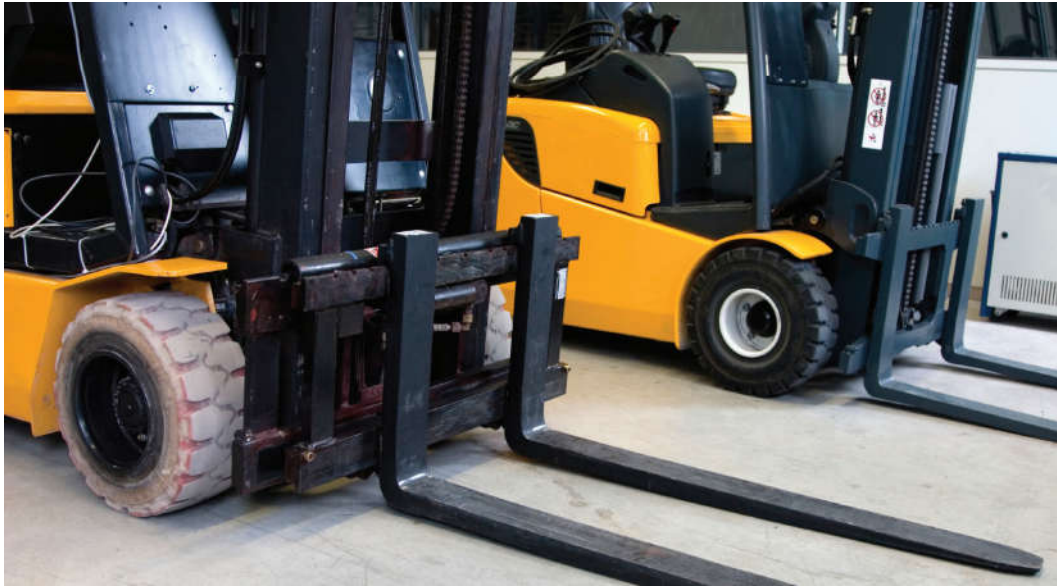


Figure 5.20 Tires are Circular

A circle (*Figure 5.21*) is defined as a closed line on a flat surface that is equidistant from a center point. There are a number of terms that we will use to describe the parts of a circle.

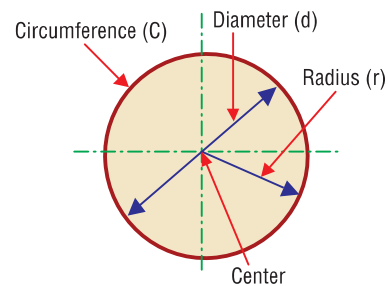


Figure 5.21 Circle Terminology

TERMINOLOGY OF CIRCLES

The **center** is a fixed point in the middle, or center, of the circle.

The **circumference** is the distance around the outside of the circle. It is measured in standard units (e.g., inches, feet, or meters) or in degrees (discussed later in this unit). All points on the circumference are the same distance (equidistant) from the center of the circle. The circumference is indicated by the letter C .

The **diameter** (Figure 5.22) is a straight line going from one side of the circle to the other and passing through the center. It divides the circle into two halves. The diameter is indicated by the letter d .

The **radius** (Figure 5.23) is a straight line beginning at the center and ending at a point on the circumference. It is indicated by the letter r . The radius is one-half of the diameter, which is represented as:

$$r = 0.5d \quad \text{radius is one-half of the diameter}$$

$$d = 2r \quad \text{diameter is twice the radius}$$

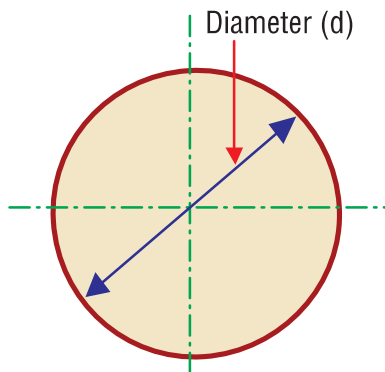


Figure 5.22 Diameter

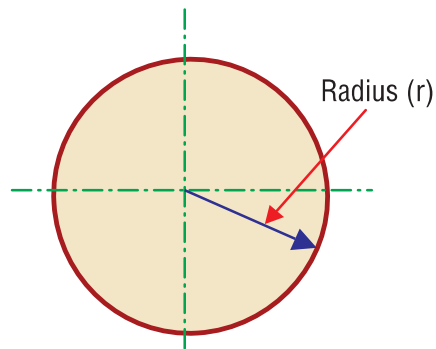


Figure 5.23 Radius

RELATIONSHIP OF CIRCUMFERENCE AND DIAMETER

In a circle, there is a special relationship between the circumference and the diameter. **The circumference of any circle is approximately 3.1416 times the size of the diameter.** This is a constant value for any size circle and is known as ***pi*** or the symbol π . The actual value of π is 3.141592654 (this number continues). Depending on the level of accuracy needed, π is often rounded to 3.14.



Fun with Numbers – Pi

Yes – π goes on forever. Here are the first several hundred places:

3.1415926535897932384626433832795028841971693993751058209749445
 923078164062862089986280348253421170679821480865132823066470938
 446095505822317253594081284811174502841027019385211055596446229
 489549303819644288109756659334461284756482337867831652712019091
 456485669234603486104543266482133936072602491412737245870066063
 155881748815209209628292540917153643678925903600113305305488204
 66521384146951941511609.....

Note: Scientific and construction calculators will have a key for π . Locate the π key. When you press this key (and, if required, the equals sign), you will see π displayed to several decimal places (Figure 5.24). When solving problems with your calculator involving π , you will not need to round π or any other numbers until you have a final answer.

CALCULATING CIRCUMFERENCE

The formula for the circumference of a circle is written as:

$$C = \pi d$$

Circumference is about 3.1416 times the diameter.

Because we have learned that $d = 2r$, we can write the previous equation as:

$$C = \pi 2r \text{ or } C = 2\pi r$$

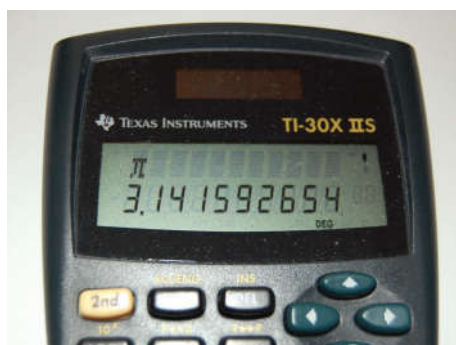


Figure 5.24 Calculator Displaying Pi

Review these examples involving circular measure.

Examples: The diameter of a circle is 15". What is the radius?

$$r = 0.5d$$

$$0.5 \times 15" = 7.5"$$

The radius of a circle is 27'. What is the diameter?

$$d = 2r$$

$$2 \times 27' = 54'$$

The diameter of a circle is 38.5". What is the circumference to two decimal places?

$$C = \pi d$$

$$\pi \times 38.5" = 120.95"$$

Solve the previous problem using a calculator:

π × 38.5 = 120.95

The radius of a circle is 4' 6 ½". What is the circumference (in feet) to two decimal places?

Note that we must first convert to feet:

6 ½" out of 12"

$6 \cdot 5/12 = 0.54'$ (to 2 places)

4' 6 ½" = 4.54

$C = 2\pi r$

$2 \times \pi \times 4.54' = 28.53'$

Answer: 240 sq ft

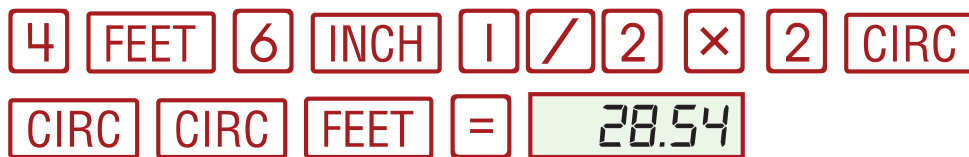
SOLVING WITH CALCULATORS

Solve this problem using a scientific calculator. Be sure that you understand why each entry is being made to solve the problem.



Note: The two answers to the problem above differ slightly. This is because the calculator does not round π .

Now, use a construction calculator to solve this problem. With the construction calculator we can enter the radius as a whole number and fraction.



When using the construction calculator, you multiply the radius times two to get the diameter. You then press the "Circle" key to determine the circumference. Because you entered the diameter as a fraction, the answer is a fraction. You can then press the "Feet" key to show the answer as a decimal fraction. Several of these steps are shown in *Figure 5.25*.



Figure 5.25 Calculating Circumference Using a Construction Calculator

ADDITIONAL TERMS TO KNOW

There are some additional terms related to a circle that are useful to learn (*Figure 5.26*).

A **segment** is the area of “part” of a circle.

An **arc** is a portion of the circumference. The straight line that connects the beginning and end of the arc is the **chord**. Note that because the diameter runs through the center of the circle, it is the longest chord.

As mentioned previously, the circumference may be measured in standard units of measure (inches, feet, or meters) or it may be measured in degrees. Measuring in degrees is known as **angular measure**.

The basic unit of angular measure is a **degree**. A circle is divided into 360 degrees, or 360° . This means that the circle is divided into 360 parts, with each part being 1 degree (*Figure 5.27*). Therefore, $1^\circ = \frac{1}{360}$ of a circle.

An **angle** (*Figure 5.28*) is the angular distance “between” two lines extending out from a single point (referred to as a **vertex**). In the angle on the left in *Figure 5.28*, the vertex is the center of the circle, and the lines are equal length and each is the radius. The angle on the right shows two lines extending out from a vertex point. The size of an angle in degrees depends on how far apart these two lines are from each other.

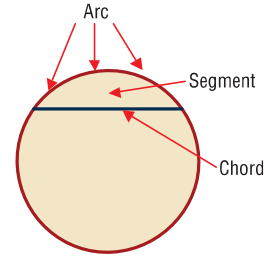


Figure 5.26 Arc, Segment, and Chord

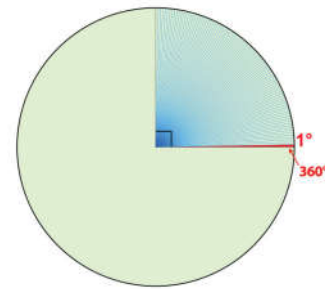


Figure 5.27 360 Degrees in a Circle

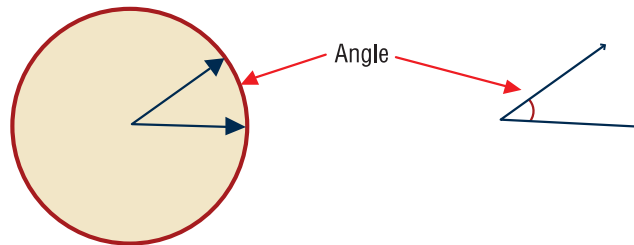


Figure 5.28 Angle

Figure 5.29 shows 360° in a circle (going counterclockwise), with the most common degrees indicated (i.e., multiples of 30, 45, and 60 degrees).

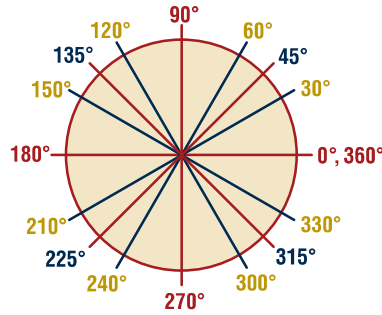


Figure 5.29 Degrees in a Circle

DIVIDING ANGLES INTO SMALLER UNITS

An angle can be divided into smaller parts or units. There are several methods for doing this; we will look at two.

CONVERTING TO DECIMAL PARTS

You can divide a degree into **decimal parts**. For example, if we have 35.5° , this means we have 35° and another “half” of a degree (or 0.5°).

CONVERTING TO MINUTES AND SECONDS

Another method of dividing a degree into smaller parts is through the use of **minutes** and **seconds**. Minutes and seconds are for very precise angular measurement. For example, when laying out a building, it may be necessary to express angular measurements in minutes and seconds.

Table 5.3 shows the relationships among the units of angular measure.

Units of Angular Measure		
1 Circumference	=	360 degrees (360°)
1 Degree	=	60 minutes ($60'$)
1 Minute	=	60 seconds ($60''$)

Table 5.3 Units of Angular Measure

Figure 5.30 shows a circle divided into four **quadrants**. There is a total of 360° in the full circle. Each fourth, or quadrant, of the circle contains 90° . An angle is created by extending two lines from the center to two points on the circumference.

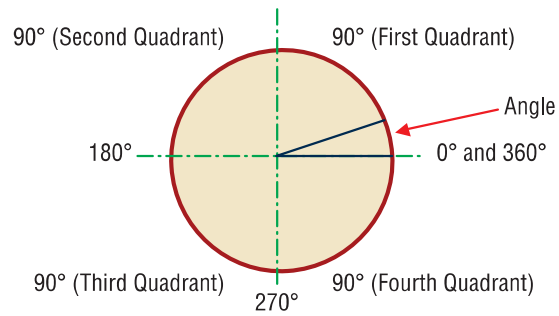


Figure 5.30 Circle Quadrants

To convert degrees to minutes, multiply the number of degrees times 60.

Convert 25° to minutes.

$$25^\circ \times 60 \text{ minutes/degree} = 1,500'$$

To convert minutes to seconds, multiply by 60.

Convert $15'$ to seconds.

$$15' \times 60 \text{ seconds/minute} = 900''$$

When performing conversions of angular measure values that include degrees, minutes, and seconds, pay particular attention to the individual conversions within the problem. Here are several examples.

Examples: Convert $4^\circ 20'$ to seconds.

Step 1: Convert 4° to minutes.

$$4^\circ \times 60 = 240'$$

Step 2: Convert minutes to seconds.

$$240' \times 60 = 14,400''$$

$$20' \times 60 = 1,200''$$

Step 3: Add both seconds values.

$$1,200'' + 14,400'' = 15,600''$$

Answer: $15,600''$

Convert 6.25° to minutes.

$$6.25^\circ \times 60 = 375'$$

Answer: $375'$

Convert 15.24° to minutes and seconds.

$$15.24^\circ \times 60 = 914.4'$$

$$0.4' \times 60 = 24''$$

$$15.24^\circ = 914'24''$$

CONVERTING DECIMAL DEGREES TO DMS BY CALCULATOR

A construction calculator will convert between decimal degrees and degrees, minutes, and seconds (DMS) automatically. Here is the sequence to convert 82.37° to DMS (Figure 5.31):

8 2 . 3 7 CONV DMS = 82.22.12

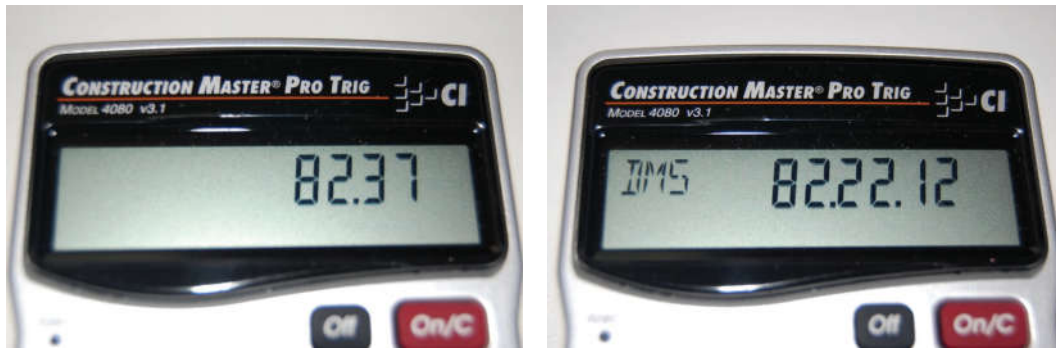


Figure 5.31 Calculating Decimal Measure to DMS Measure

CONVERTING DECIMAL DEGREES TO DMS MANUALLY

Following are the steps to convert from decimal degrees to DMS without a calculator. For example, as in the previous problem, convert 82.37 degrees to DMS.

Example: Convert 82.37 degrees to DMS.

Step 1: The whole units of degrees will remain the same.
In this example, that is 82° .

Step 2: Multiply the decimal by 60.
 $0.37 \times 60 = 22.2$

Step 3: The whole number becomes the minutes. Whole number
minutes = $22'$

Step 4: Take the remaining decimal, and multiply by 60.
 $0.2 \times 60 = 12$

Step 5: The resulting number becomes the seconds (12"). Seconds can remain as a decimal (although in this problem, the number or seconds turned out to be a whole number).

Step 6: Take your three sets of numbers, and put them together using the symbols for degrees ($^{\circ}$), minutes ($'$), and seconds ($"$). The result is $82^{\circ}22'12"$.

CONVERTING FROM DMS TO DECIMAL EQUIVALENT

Now, let's look at the steps to convert from DMS to a decimal equivalent.

Example: Using the problem above, convert $82^{\circ}22'12"$ to a decimal equivalent.

Remember that each second is $1/60$ th of a minute, and each minute is $1/60$ th of a degree. This means that each second is $1/60$ th of $1/60$ th, or $1/60 \times 1/60 = 1/3600$. So, the conversion looks like this:

$$82^{\circ} + (22 \times 1/60) + (12 \times 1/3600) = 82.37^{\circ}$$

Now you should be glad you have calculators that will do these conversions automatically!

CALCULATING AREA OF A CIRCLE

We know that to calculate the area of a square or rectangle, we multiply the length times the width. So, how do you calculate the area of a circle? Assume you have a round metal plate and need to know the area. There is no "length" or "width" – instead there is a circumference that is the same distance from the center of the circle.

To calculate the area of a circle, use this formula:

$$A = \pi r^2 = \pi \times r \times r$$

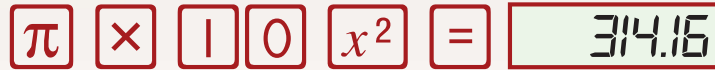
This means that you multiply pi (π) times the radius squared. Remember that the radius is the distance from the center of the circle to a point on the circumference of the circle. Also note that the radius is one-half of the diameter (the distance from one side of the circle to the other passing through the center of the circle).

Let's look at some sample problems.

Example: What is the area of a circle that has a radius of 10"?

$$A = \pi r^2 = \pi \times 10'' \times 10'' = 314.16 \text{ sq in}$$

Here is the sequence to solve this problem using a scientific calculator:



A sequence of calculator buttons: π , \times , 10, x^2 , =, and a display showing 314.16.

Here is the sequence to solve this problem using a construction calculator. Note that we must first multiply the radius times 2 to determine the diameter.



A sequence of calculator buttons: 10, \times , 2, CIRC, CIRC, =, and a display showing 314.16.

Example: What is the area of a circle that has a radius of 12'9"?

$$A = \pi r^2$$
$$\pi \times 12.75' \times 12.75' = 510.71 \text{ sq ft}$$

SUMMARY OF FORMULAS

Following is a summary of the formulas used in circular measure.

$r = 0.5d$	Radius is one-half of the diameter.
$d = 2r$	Diameter is twice the radius.
$C = \pi d$	Circumference is 3.1416 times the diameter.
$C = 2\pi r$	Circumference is 2π times the radius.
$A = \pi r^2$	Area of a circle is π times the radius squared.
1 circle = 360°	
1 degree = 60 minutes (60')	
1 minute = 60 seconds (60'')	

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 4

VOLUME MEASURE

The previous content in this unit dealt with calculating linear and surface (area) measure. There are, however, times when BAC Craftworkers need to calculate the “volume” of a container or the amount of space, measured in cubic units, that an object or substance occupies.

We've learned that the area of a square or rectangle is calculated by multiplying the length times the width. When the length and width are in inches, the area is in square inches. For feet, the area is in square feet. For meters, the area is in square meters.

DETERMINING VOLUME

One square inch is two-dimensional. When we add depth to the square inch, we have **volume**. If we add 1" in depth to the square inch, we have 1 cubic inch. The cubic inch (or **cubic solid**) is the basic unit of volume measure in the English system. *Figure 5.32* shows 1 cubic inch.

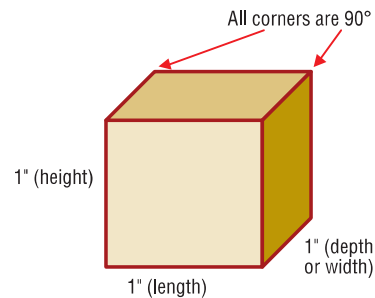


Figure 5.32 Cubic Inch

When the length, height, and depth all equal 1 foot, the volume is 1 cubic foot. Because there are 12 inches in 1 foot, $12 \times 12 \times 12 = 1,728$ cubic inches in 1 cubic foot.

If all three dimensions equal 1 yard, the volume is 1 cubic yard (as in a cubic yard of concrete). Because there are 3' in 1 yard, there are $3 \times 3 \times 3 = 27$ cubic feet in 1 cubic yard.

Table 5.4 shows the basic units of volume measure.

Table of Volume Measure		
Basic unit of measure	=	1 cubic inch (cu in)
1,728 cu in	=	1 cubic foot (cu ft)
27 cu ft	=	1 cubic yard (cu yd)

Table 5.4 Units of Volume Measure

CONVERTING UNITS OF MEASURE

Using the values in *Table 5.4*, you can convert from one unit of measure to another. Review the following examples.

Examples: Convert 2.5 cubic feet to cubic inches.

$$2.5 \text{ cu ft} \times 1,728 \text{ cu in/cu ft} = 4,320 \text{ cu in}$$

Convert 15,000 cubic inches to cubic feet, accurate to 2 places.

$$15,000 \text{ cu in} \div 1,728 \text{ cu in/cu ft} = 8.68 \text{ cu ft}$$

Convert 1.75 cubic yards to cubic inches.

Step 1: Convert to cubic feet:

$$1.75 \text{ cu yd} \times 27 \text{ cu ft/cu yd} = 47.25 \text{ cu ft}$$

Step 2: Convert cubic feet to cubic inches:

$$47.25 \text{ cu ft} \times 1,728 \text{ cu in/cu ft} = 81,648 \text{ cu in}$$

Answer: 81,648 cu in

CONVERTING UNITS WITH CALCULATORS

Some construction calculators will perform these conversions automatically. For example, following is the sequence for how one construction calculator could solve the conversion of 1.75 cubic yards to cubic inches. Note that the "Yards" key (Yds) is pressed 3 times to tell the calculator that you are working with cubic yards.



DETERMINING WEIGHTS OF MATERIAL

Many times, the Craftworker will need to determine the weight of a piece of material. In fact, these calculations are so common that there are tables that show weights of common materials. *Table 5.5* presents plate thickness and weight in pounds per square foot. For example, note that a 1" plate weighs approximately 40.84 pounds per square foot.

THEORETICAL PLATE WEIGHTS AND U.M. PLATE WEIGHTS BASED ON DENSITY FACTOR OF .28361 # PER CUBIC INCH	
Plate Thickness (Inches)	Pounds per Square Foot
3/16	7.66
1/4	10.21
5/16	12.76
3/8	15.32
7/16	17.87
1/2	20.42
9/16	22.97
5/8	25.53
3/4	30.63
7/8	35.74
1	40.84
1 1/8	45.95
1 1/4	51.05
1 3/8	56.16
1 1/2	61.26
1 5/8	66.37
1 3/4	71.47
2	81.68
2 1/4	91.89
2 1/2	102.10
2 3/4	112.31
3	122.52
3 1/4	132.73
3 1/2	142.94
3 3/4	153.15
4	163.36
4 1/4	173.57
4 1/2	183.78
4 3/4	193.99
5	204.20
5 1/2	224.62
6	245.03

Table 5.5 Plate Thickness and Weight

Using the information in *Table 5.5*, determine the weight of a 1" metal plate, measuring 3 feet by 3 feet:

Example: Determine area of the plate

$$3' \times 3' = 9 \text{ sq ft}$$

Step 1: Consult table for weight

1" plate weighs 40.84 lbs/sq ft.

Step 2: Multiply area times weight

$$9 \text{ sq ft} \times 40.84 \text{ lbs/sq ft} = 367.56 \text{ pounds}$$

Answer: 367.56 lbs

WHAT IS VOLUME?

What does “volume” contain? One cubic foot might be a solid (e.g., metal, wood, concrete), a liquid (e.g., water, oil, solvent), or simply air or open space. Volume may also be a combination of these.

Although there are several **units of liquid measure**, the most common in terms of BAC Craftworking in the English system of measure are the pint, quart, and gallon as shown in *Table 5.6*.

UNITS OF LIQUID MEASURE		
2 pints (pt)	=	1 quart (qt)
4 quarts	=	1 gallon (gal)
1 gallon	=	231 cubic inches

Table 5.6 Units of Liquid Measure

Use Table 5.6 with the following examples to convert liquid measurements.

Examples: Convert 8.5 pints to quarts.

$$8.5 \text{ pt} \div 2 \text{ pt/qt} = 4.25 \text{ qt}$$

Convert 3.75 gallons to quarts.

$$3.75 \text{ gal} \times 4 \text{ qt/gal} = 15 \text{ qts}$$

How many gallons of solvent are equivalent to 15,000 cubic inches?

$$15,000 \text{ cu in} \div 231 \text{ cu in/gal} = 64.94 \text{ gallons (to 2 places)}$$

VOLUME OF RECTANGULAR SOLIDS

Cubic volume applies not only to cubes but also to **rectangular solids** (Figure 5.33). To calculate the volume of a cubic or rectangular solid, multiply the length times the width times the height. As with calculating areas, all values must be in the same unit of measure (e.g., inches, feet, yards, or meters). The formula is written:

$$V = l \times w \times h$$

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Review the following examples.

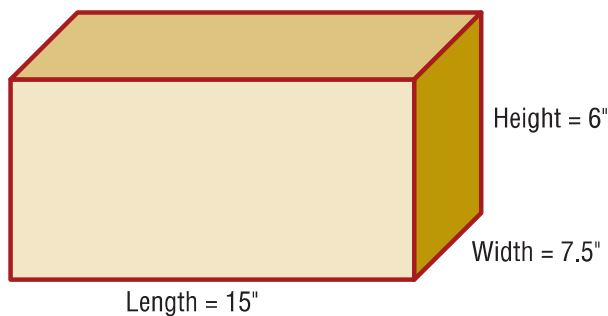


Figure 5.33 Volume of a Rectangle

Examples: Calculate the volume of a rectangular solid that measures 15" by 7.5" by 6" (Figure 5.33).

$$\begin{aligned} V &= l \times w \times h \\ &= 15" \times 7.5" \times 6" \\ &= 675 \text{ cu in} \end{aligned}$$

Calculate the volume of a tank that measures 6.5' by 3' and is 2.5' high.

$$V = l \times w \times h$$
$$6.5' \times 3' \times 2.5' = 48.75 \text{ cu ft}$$

Concrete is needed to fill a rectangular space. The space is 18' long, 4.5' wide, and 7' deep. How many cubic yards of concrete are required to fill the space?

Step 1: Calculate volume in measurements provided (cubic feet)

$$V = l \times w \times h$$
$$18' \times 4.5' \times 7' = 567 \text{ cu ft}$$

Step 2: Convert cubic feet to cubic yards

$$567 \text{ cu ft} \div 27 \text{ cu ft/cu yd} = 21 \text{ cu yd}$$

Answer: 21 cu yd

A square fuel container measures 3.5' tall with a base that measures 2' on one side. How many gallons of fuel will the container hold?

Step 1: Calculate the volume in cubic inches

$$V = l \times w \times h$$
$$24" \times 24" \times 42" = 24,192 \text{ cu in}$$

Step 2: Convert the volume to gallons

$$24,192 \text{ cu in} \div 231 \text{ cu in/gal} = 104.73 \text{ gal (to 2 places)}$$

Answer: 104.73 gal

DETERMINING VOLUME OF A CYLINDER

A circle with depth becomes a **cylinder** (Figure 5.34). The formula to calculate the volume of a cylinder is:

$$V = \pi r^2 h$$

Volume = pi \times radius squared \times height

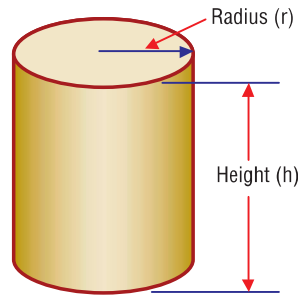


Figure 5.34 Cylinder

Example: A cylinder tank has a radius of 12" and a height of 36". What is the volume of the tank to the nearest cubic inch?

$$V = \pi r^2 h$$

$$\pi \times 12^2 \times 36 = 16,286 \text{ cu in}$$

Use a scientific calculator to solve the previous problem.



Example: A large water storage tank has a diameter of 24' and height of 18'. If the tank is half full, how many gallons of water are in the tank?

Step 1: Calculate total volume of the whole tank in given measure (cubic feet).

$$V = \pi r^2 h = \pi \times 12^2 \times 18 = 8,143 \text{ cu ft}$$

(to the nearest cu ft)

Step 2: The tank is half full; determine half of the tank's total volume.

$$8,143 \text{ cu ft} \times 0.5 = 4,071.5 \text{ cu ft of water}$$

Step 3: Convert to cubic inches.

$$4,071.5 \text{ cu ft} \times 1,728 \text{ cu in/cu ft} = 7,035,552 \text{ cu in}$$

Step 4: Convert to gallons.

$$7,035,552 \text{ cu in} \div 231 \text{ cu in/gal} = 30,456.94 \text{ gal}$$

(to 2 places)

Answer: 30,456.94 gal

Note: The summary table (*Table 5.7*) of conversions and formulas will help you complete the Assignment Sheets and prepare for the Unit Test. Your instructor will advise you whether you will be able to use this summary table during the Unit Test or if you will need to memorize this information.

CONVERSIONS AND FORMULAS	
Description	Conversion or Formula
Linear measure	1 foot = 12 inches (12") 1 yard = 3 feet (3') 1 yard = 36 inches (36")
Perimeter (distance around the outside of a shape)	Perimeter = sum of the lengths of each side or $P = 2L + 2W$
Area of a rectangle	$A = l \times w$ (Area = length \times width)
Area of a square	$A = s^2$ (Area = length of 1 side squared)
Area of a triangle	$A = 1/2bh$ (Area = $1/2 \times$ base \times height)
Area of a parallelogram	$A = l \times w$ (Area = length \times width)
Area of a trapezoid	$A = (b_1 + b_2) \times 0.5h$ (Area = sum of the 2 bases times one-half of the height)
Radius and diameter	$r = 0.5d$ (Radius is one-half of the diameter)
Diameter and radius	$d = 2r$ (Diameter is twice the radius)
Circumference	$C = \pi d$ (Circumference is π or about 3.1416 times the diameter)
Circumference	$C = 2\pi r$ (Circumference is π or about 3.1416 times twice the radius)
Angular measure	1 circumference = 360 degrees (3600) 1 degree = 60 minutes (60') 1 minute = 60 seconds (60")
Area of a circle	$A = \pi r^2$ (Area = π or about 3.1416 times the radius squared)

Volume measure	Basic unit = 1 cubic inch (cu in) 1,728 cu in = 1 cubic foot (cu ft) 27 cu ft = 1 cubic yard (cu yd)
Liquid measure	2 pints = 1 quart (qt) 4 qt = 1 gallon (gal) 1 gal = 231 cu in
Volume of a cubic or rectangular solid	$V = l \times w \times h$ (Volume = length \times width \times height)
Volume of a cylinder	$V = \pi r^2 h$ (Volume = $\pi \times$ radius squared \times height)

Table 5.7 Conversions and Formulas

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

GLOSSARY

Abacus (also called a *counting frame*): A calculating tool used primarily in parts of Asia for performing arithmetic processes.

Acute angle: An angle measuring less than 90° .

Acute triangle: A type of triangle where all three of the interior angles are less than 90° .

Addition: The process of combining two or more numbers into one number; the opposite, or inverse, of subtraction.

Adjacent angles: Two angles that share a common vertex and side but do not overlap.

Alternate exterior angles: The angles created when two lines are crossed by another line; the pairs of angles on opposite sides of the transversal but outside the two lines.

Alternate interior angles: The angles created when a transversal crosses two (usually parallel) lines; the pairs of angles inside the parallel lines and on opposite sides of the transversal.

Angle: The angular distance “between” two lines extending out from a single point (referred to as a vertex).

Angular measure: Measurement in degrees.

Arc: A portion of the circumference.

Area (Surface): The measurement of something that has width and length (or width and height).

Average: The number that best represents all numbers in a series. It is found by adding all numbers in the series and then dividing that total by the number of numbers in the series.

Base: In multiplication, the number that is to be multiplied in an exponential equation. In trigonometry, the bottom side of a triangle.

Center: The fixed point in the middle of a circle.

Centimeter: A metric unit of length equal to one hundredth of a meter.

Chord: A straight line that connects the beginning and end of an arc.

Circle: A closed line on a flat surface that is equidistant from a center point.

Circumference: The distance around the outside of the circle, measured in standard units (e.g., inches, feet, or meters) or in degrees.

Common fractions: A type of fraction in which one number is written over another number. The top number in a common fraction is called the numerator, and the bottom number is called the denominator.

Complementary angles: When two angles equal 90° if added together.

Construction calculator: A type of calculator that typically allows for the user to easily determine precise angle measurements and solve complex design and construction-math problems (including trigonometric functions, 3-4-5 right triangles, and estimating materials and costs).

Corresponding angles: When two lines are crossed by another line (which is called the transversal), the angles in matching corners.

Cosine (abbreviated as *cos*): The ratio of the side adjacent to the hypotenuse.

Cross-multiplication: To multiply the numerator of one pair of fractions by the denominator of the other.

Cubic foot: The volume equal to a cube one foot on each side.

Cubic inch (also known as *cubic solid*): The basic unit of volume measure in the English system.

Cubic yard: A unit of volume ($1 \text{ yd} \times 1 \text{ yd} \times 1 \text{ yd}$).

Cylinder: A circle with depth.

Decimal fraction: Any number written in the form: an integer followed by a decimal point followed by a (possibly infinite) string of digits.

Decimal point: The symbol in a decimal number used as a reference point for place value.

Decimal system: A numeration system based on powers of 10.

Decimeter: One tenth of a meter (metre).

Degree: The basic unit of angular measure.

Denominator: The bottom number in a fraction.

Dekameter: Ten meters (almost 11 yards).

Diagonal(s): A straight line inside a shape that goes from one corner to another.

Difference: The answer that results when subtracting one number from another.

Diameter: The straight line that travels from one side of the circle to the other, passing through the center. It divides the circle into two halves.

Digits: A single character in a numbering system. In our numbering system, there are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Dividend: A number that is to be divided by a divisor.

Divisible: Any number that is capable of being divided, especially with no remainder.

Division: The process of determining how many times one number is contained in another number.

Divisor: The number being divided into the dividend in a division problem.

Endpoint(s): Either of the two points marking the end of a line segment.

Equals sign (=): Sign used to indicate the sum of the numbers being added.

Equilateral triangle: A special type of triangle in which the three sides are equal in length, and, therefore, the three interior angles are also equal in length. Given that the interior angles must total 180° , all three interior angles in an equilateral triangle equal 60° .

Exponent: The number indicating how many times the number in the base is to be multiplied.

Extremes: The first and last terms in a proportion.

Fraction: A number that expresses part of a whole. It is written in the form of a/b or $a \div b$ (where both a and b are whole numbers, and the number b is not 0).

Geometry: The study of the size, shape, and position of two-dimensional shapes and three-dimensional figures.

Gram: The basic metric unit of weight or mass.

Hectometer: 100 meters (almost 110 yards).

Height: The extent or distance upward.
Example: The building has a height of 500 feet.

Hypotenuse: The side of a right triangle opposite the right angle; the longest side of a right triangle.

Improper fraction: A type of fraction in which the numerator is equal to or greater than the denominator.

Isosceles triangle: A special type of triangle in which two of the three sides are equal in length, and the two opposite interior angles are also equal.

Kilometer: 1 000 meters (about $\frac{5}{8}$ of a mile).

Line: A geometrical object that is (usually) straight, infinitely long and infinitely thin. A line has no width or depth but will have length.

Linear: Something that is in line or straight.

Linear measure (also known as *measurement of length*): A type of measurement involving straight-line distances between two points.

Line segment: A line linking two points; a connected piece of a line.

Liter: The basic metric unit of volume.

Lowest Common Denominator (LCD): The least common multiple of the denominators of a set of fractions.

Major diagonal: The distance between the furthest opposite corners of the parallelogram.

Means: The second and third terms in a proportion.

Meter (Also spelled metre.): The basic metric unit of length and distance; measures about 39.27 inches.

Metric system (also known as the *International System of Units, or SI [Système International d'Unités in French]*): International decimal-based system of measurement that is most commonly used throughout the world. It includes standards for many types of measures, including: length (millimeter, centimeter, meter, kilometer), volume (millimeter, cubic centimeter, liter, cubic meter), mass or weight (milligram, gram, kilogram, metric ton), and temperature (centigrade).

Midpoint: A point on a line segment that divides it into two equal parts.

Millimeter: A unit of length equal to one thousandth (0.001) of a meter.

Minor diagonal: The distance between the closest opposite corners of the parallelogram.

Minuend: The number from which the subtrahend is subtracted.

Minus sign (-): A sign used to indicate that one number is to be subtracted from another number.

Minutes: The sixtieth part of a degree of angular measure, often represented by the sign “'”, as in $12^{\circ} 10'$, which is read as 12 degrees and 10 minutes.

Mixed number: An improper fraction that is expressed as a combination of a whole number and a fraction.

Multiplicand: The number in a multiplication problem that is or is to be multiplied by another. For example, in the equation 8×4 , 4 is the multiplicand.)

Multiplier: The number in a multiplication problem that is multiplied. For example, in the equation 5×6 , 5 is the multiplier.

Multiplication: The process of finding the number or quantity (product) obtained by repeated additions of a specified number or quantity (multiplicand) a specified number of times (multiplier).

Numerator: The top number in a fraction.

Oblique triangle: A type of triangle that does not contain a right angle. All three angles may be acute (less than 90°), or this type of triangle may contain one angle that is obtuse (more than 90°).

Obtuse angle: An angle measuring more than 90° (but less than 180°).

Obtuse triangle: A type of triangle where one of the interior angles is greater than 90° .

Opposite angles: The two angles that, when two lines intersect and four angles are formed, are directly opposite to each other.

Parallel: When two lines do not intersect at any point (e.g., railroad tracks are parallel; two columns in a structure are usually designed to be parallel.)

Parallelogram: A figure or shape within which the two lengths are parallel, and the two heights (or widths) are parallel.

Percent (Percentage): “Per one hundred,” or the number of parts per 100. As its meaning suggests, percent is a special kind of fraction, one which is always represented with the number 100 in the denominator.

Perimeter: The distance around a two-dimensional shape.

Perpendicular: Intersecting at or forming right angles.

Place value: Where the number is located in relation to the decimal point.

Plane geometry: The study of geometry involving points and lines on flat surfaces.

Plus sign (+): A sign used to indicate that two or more numbers are to be added.

Point: An infinitely small sphere with no length, width, or height; a location on the surface.

Prime number: A number only divisible by itself and 1. Examples of prime numbers include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc.

Product: The number that is determined after two or more numbers have been multiplied.

Proper fraction: A form of division in which the result has a value smaller than 1.

Proportion: A statement of equality between two ratios.

Pythagorean Theorem: The mathematical theorem, discovered by ancient Greek mathematician Pythagoras, stating that the square of the hypotenuse to a triangle is equal to the sum of the squares of the other two sides. ($C^2 = A^2 + B^2$)

Quadrant: 90° , or a fourth of a circle (360°).

Quadrilateral: Any four-sided figure.

Quotient: The number obtained by dividing one quantity by another. For example, in the equation $45 \div 3 = 15$, 15 is the quotient.

Radian(s): A unit of plane angle equal to $180/\pi$ (or $360/2\pi$) degrees, or about 57.2958 degrees, or about $57^\circ 17' 45''$; the standard unit of angular measurement in all areas of mathematics beyond the elementary level.

Radius: A straight line beginning at the center of a circle and ending at a point on the circumference.

Ratio: The indicated quotient of two mathematical equations; the relationship in quantity, amount, or size between two things; proportion.

Rectangle: A shape having four sides. The two lengths in a rectangle are parallel and the same, and the two widths are parallel and the same.

Reflex angle: An angle that measures more than 180° .

Remainder: The number(s) “left over” after dividing two numbers.

Rhombus: A parallelogram with four equal sides; an oblique-angled equilateral parallelogram.

Right angle: An angle that measures 90° .

Right triangle: A type of triangle that contains a 90° angle.

Scientific calculator: A type of calculator, usually but not always handheld, designed to calculate problems in science, engineering, and mathematics.

Seconds: Each minute is split up into 60 parts, each part being $\frac{1}{60}$ of a minute. These parts are called seconds.

Segment: A part cut off from a figure, especially a circular or spherical one, by a line or plane, as a part of a circular area contained by an arc and its chord or by two parallel lines or planes.

Sine (abbreviated as *sin*): The ratio of the side opposite to the hypotenuse.

Slide rule (also known as a *slipstick*): A mechanical calculator used primarily for multiplication, division, and “scientific” functions, such as roots, logarithms, and trigonometry.

Square: In multiplication, a number that is multiplied by itself. In geometry, a shape with all four sides being of equal length. All four angles within a square are 90° .

- Square inch:** A unit of area measurement equal to a square measuring one inch one each side; 6.452 square centimeters.
- Square root:** A number that when multiplied by itself equals a given number. For example, the square root of 64 is represented by the “ $\sqrt{64}$ ” symbol.
- Square unit:** The measurement label given to measurements of area. Identifies the two-dimensions present.
- Straight angle:** An angle that measures 180° (a straight line).
- Subtraction:** The mathematical process of finding the difference between two numbers; the inverse, or opposite, of addition.
- Subtrahend:** The quantity or number to be subtracted from the minuend. For example, in the example $86 - 16$, 16 is the subtrahend.
- Sum:** The number that results when two or more numbers have been added together.
- Supplementary angles:** When two angles equal 180° when added together.
- Tangent (abbreviated as *tan*):** The ratio of the side opposite to the side adjacent.
- Times sign (\times):** Sign used to indicate that one number is to be multiplied by another number.
- Trapezoid:** A four-sided figure or shape containing two sides (called *bases*) that are parallel.
- Triangle:** A type of shape having three sides and three interior angles.
- Trigonometry:** The branch of mathematics that deals with the relations between the sides and angles of plane or spherical triangles and the calculations based on them.
- United States Customary System of units of measurement (also called the *English, Imperial, American, or inch system*):** The primary and most commonly used system of measurement in the United States having standards for many types of measures, including: length (inch, foot, yard, mile), volume (cubic inch, cubic foot, cubic yard), mass or weight (ounce, pound, ton), and temperature (Fahrenheit).
- Vertex:** A corner point of a triangle or other geometric figure.
- Volume:** The amount of three-dimensional space an object occupies.
- Weight:** The amount or quantity of heaviness or mass; amount a thing weighs.
- Whole numbers:** Complete numbers with no fractional parts of the number left over. For example, 50 is a whole number.

