CONSTRUCTION MATHEMATICS FOR CRAFTWORKERS

STUDENT MANUAL

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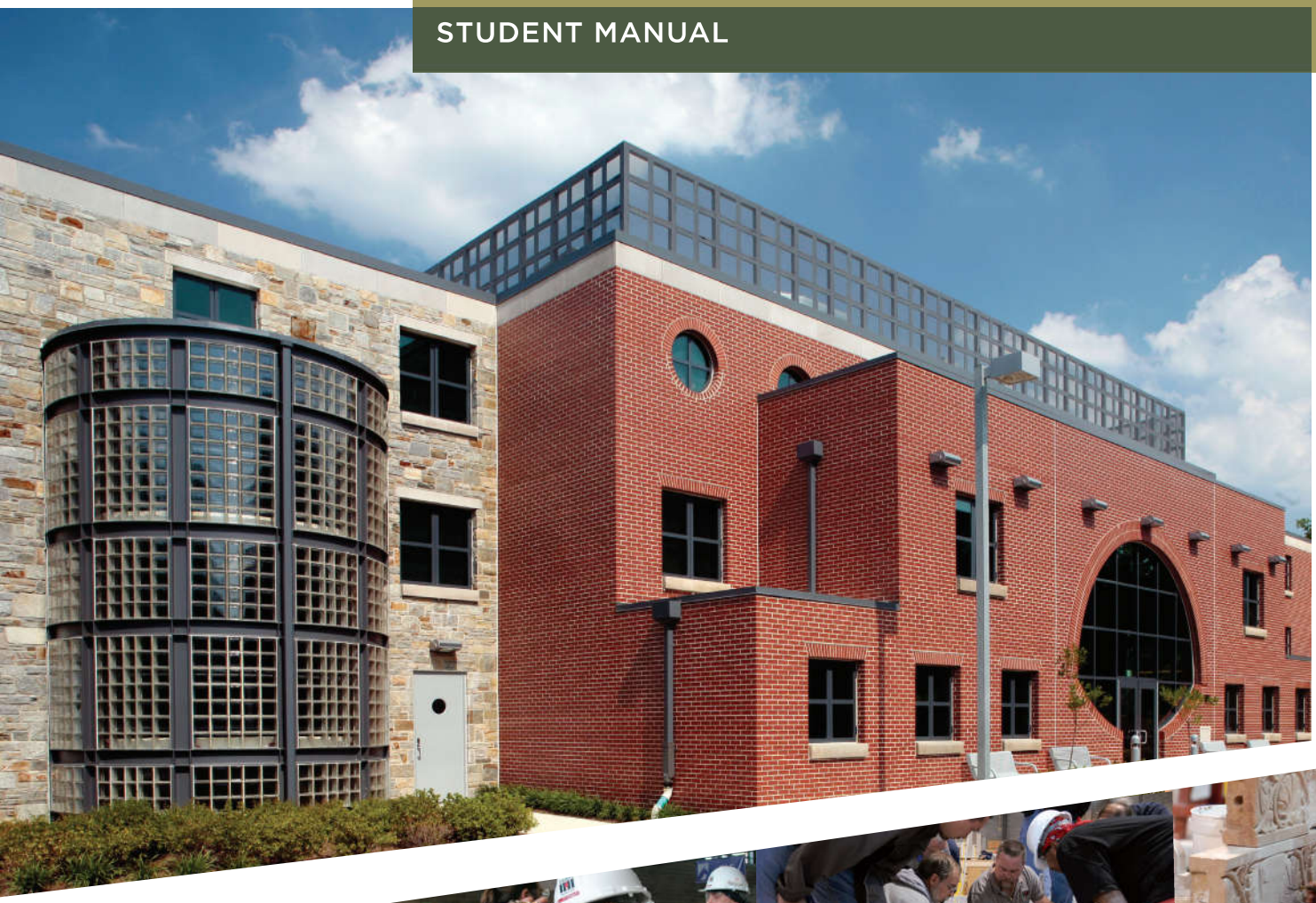
INTERNATIONAL UNION OF BRICKLAYERS AND ALLIED CRAFTWORKERS
and the IRONWORKERS

BASIC MATHEMATICS FOR BAC CRAFTWORKERS

STUDENT MANUAL



INTERNATIONAL UNION OF BRICKLAYERS AND ALLIED CRAFTWORKERS

BRICK | STONE | TILE | MARBLE | TERRAZZO | PLASTER | CEMENT | RESTORATION

BAC Craftworker Quality Construction Practices

Basic Mathematics for BAC Craftworkers

Student Manual

International Union of Bricklayers and Allied Craftworkers

APPRENTICESHIP AND TRAINING DEPARTMENT AND NATIONAL FUND MISSION STATEMENT

The International Masonry Institute (IMI) and the International Masonry Training and Education Foundation (IMTEF) are joint labor/management organizations created through the leadership of the International Union of Bricklayers and Allied Craftworkers (BAC) and the contractors who employ BAC members. IMI works to expand and improve business opportunities for mason contractors as well as job opportunities and working conditions for BAC members. IMI promotes the quality of union-built masonry to architects, engineers, building owners, public officials, and the general public throughout the United States and Canada. IMTEF provides training to apprentices, and advanced and specialized training to journeyworkers, supervisors, and trade instructors.

These manuals are provided to assist in your training and educate you concerning the importance of safety on the job. It is important for you to fully understand that your employer and other employers on the jobsite are first and foremost responsible for creating a safe environment. The IMTEF is not responsible for the formulation of employer safety policies or maintenance of safety on the jobsite.

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CONTRACTORS & CONSULTANTS

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ACKNOWLEDGEMENTS

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Organizations

We would like to acknowledge the International Association of Bridge, Structural, Ornamental and Reinforcing Iron Workers for their permission to use selected content in the development of this manual.

PREFACE

TRAINING PACKAGE OVERVIEW

This training package is designed for use by both the IMTEF instructor and student. There are four primary components of this package: the Student Manual, Student Workbook, Electronic Presentation, and Instructor Guide. The student needs to understand the purpose of the first three of these components for a successful training experience. Following is a brief description of each.

COURSE MATERIALS

Student Manual – The Student Manual is the heart of the training package and contains all of the information the BAC Craftworker student needs to learn during this course. The student will read the manual as assigned. The information is presented in a unit of instruction format. To support the content presented in the Student Manual, callout notes, photographs, illustrations, details, and other information are included. There is also a glossary defining key terms from the units. When first mentioned, glossary words appear in bold in the Student Manual.

Student Workbook – Students may receive a Student Workbook and/or the instructor may copy or print required Assignment Sheets and Skill Sheets for students from the Electronic Presentation's resources. The workbook contents are for students to use before and during classroom sessions. The Assignment Sheets may be assigned as homework or in-class activities for individuals or small groups of students.

Electronic Presentation – The instructor will use the presentation to enhance student understanding and facilitate engagement during classroom instruction. The electronic presentation will support the instructor's delivery of Student Manual content primarily. The presentation may also include informative and instructional videos and additional resources not provided in the manual. Supplemental materials, such as trade information and resources, SDSs, and course handouts, may be accessible to the instructor from the presentation interface when relevant.

A student who fully engages in the reading, workbook assignments and activities, and classroom instruction should be successful in this BAC Craftworker training.

Instructor Guide – For instructor use only.

COURSE ORGANIZATION

Mathematics for BAC Craftworkers is broken into two sections: *Basic Mathematics* and *Advanced Mathematics*. This Student Manual covers the units presented in *Basic Mathematics*.

COURSE SYLLABUS

For each course there is a course syllabus. A syllabus serves as the design document for a course, provides information about the course, and is typically given to students on the first day of class. A syllabus may include the following information:

- Course title and description
- Course objectives
- Description of the target audience and course prerequisites
- Length and dates of the course
- Location of the course
- Information about the instructors
- Description of materials the student will need and/or receive
- Description of the course assignments
- Description of the course grading criteria
- Attendance criteria

UNIT OF INSTRUCTION FORMAT

The Student Manual includes a number of units of instruction. Although the various components of a unit of instruction are separated and will appear in the Student Manual, Instructor Guide, and Student Workbook, they are considered to be part of an overall package of instruction. Each unit of instruction typically includes objectives and Assignment Sheets, tests, and (for instructors) answers to the Assignment Sheets and tests.

OBJECTIVES AND SUPPORTING MATERIAL

Each unit of instruction is based on performance objectives with clear learning outcomes. These objectives state the goals of the unit, to provide a sense of direction and accomplishment for the student.

Performance objectives are stated in two forms: a unit objective, stating the subject matter to be covered in the unit, and specific objectives, stating the student performance necessary to reach the unit objective.

Objectives for each unit provide direction for the learning process, so it is important for the instructor and students to have a common understanding of the intent of the objectives. Given the importance of these objectives, they will appear in the Student Manual, Instructor Guide, and Student Workbook.

In the Student Manual, each objective is supported by a major topic within the unit, explaining and expanding on the objective. These major topics parallel the objectives as listed at the opening of the unit. If there are 10 objectives, there will be 10 major topics covered in the same order as the objectives.

ASSIGNMENT SHEETS

Assignment Sheets give direction to study and furnish practice for paper and pencil activities to develop the knowledge that is a necessary prerequisite to skill development. Your instructor may assign these for completion in class or as outside assignments. The Assignment Sheets are in the Student Workbook.

KNOWLEDGE AND SKILL TESTS

Tests have been constructed to measure achievement of each objective listed in the units. Testing will help to determine if students have achieved the unit and specific objectives.

USING THIS STUDENT MANUAL BEFORE COURSE SESSIONS

This Student Manual contains the objectives and information sheets of the unit of instruction. The student should look at this manual as the "textbook" for the course. Here are some suggestions for using this manual before course sessions.

1. Refer to the course syllabus to determine which unit or units will be covered during a specific course session. Also note when any homework assignments are due (see the Student Workbook).

2. Review the unit and specific objectives. Note that the Unit Tests will include some items related to each specific objective. So if you know the content related to each objective, you will do better on the test.

3. Read the content in the manual, being sure to study all photographs, drawings, examples, and other materials. If permissible, make notes on the pages of the manual of any questions or comments you have regarding the content.

4. Bring your manual and Student Workbook to each course session.

USING THIS STUDENT MANUAL DURING COURSE SESSIONS

During course presentations you will use the Student Manual in several ways.

1. Follow along as the instructor presents the information in the unit. The instructor will be using a computer and projector to present key points.

2. If permissible, make notes on the pages and highlight key points, answers to questions, and so on.

3. Refer to the manual when working individually or in small groups on Assignment Sheets or other classroom activities.

STUDYING FOR TESTS

After the class has completed a unit of instruction, the instructor may administer a Unit Test. The test only covers the information contained within the unit and is based on the specific objectives. If you study the information related to each objective, you will be ready for the test.

Here are some tips for preparing for a test:

- Study in a place that is free of distractions. Have ready all the things you will need, such as your Student Manual, paper, pencils, or a calculator.
- Study at a time when you are alert and not hungry or sleepy.
- Do not wait until the last minute to study! Short, daily study sessions are better than one long session the night before the test.
- Set a goal for each study period. Review one unit at a time to prepare for the test.
- Repetition is key! Read and reread your manual and review your notes.
- While you are reviewing your notes, cover them up periodically and summarize them out loud. Pretend that you are explaining the material to someone else. Better yet – do this with another student.
- Study with another student. Close your book and have the other student ask you questions. Then you ask the questions.

UNIT 1

INTRODUCTION TO MATHEMATICS

OBJECTIVES

After completion of this unit, you should be able to describe the basic history of mathematics, identify the types of number systems, and select a calculator to use for solving mathematics problems. This knowledge will be evidenced by correctly completing the Assignment Sheet and by scoring a minimum of 70% on the Unit Test.

Specifically, you should be able to:

1. Describe the basic history of mathematics.
2. Identify two basic systems of measurement used by BAC Craftworkers.
3. Select a calculator.
4. Use the Student Manual to learn mathematics required for BAC Craftworkers.

Each of these objectives is covered in the pages that follow.

OBJECTIVE 1

HISTORY OF MATHEMATICS

UNIT 1

Mathematics is the study of the measurement, properties, and relationships of quantities using numbers (e.g., 1, 2, 3½, 145.67, etc.) and symbols (e.g., +, –, ×, ÷, π, etc.). Do BAC Craftworkers use mathematics? Yes!

Figure 1.1 Architect Design Drawings and Calculator

WHO USES MATH?

Engineers and Architects use a great deal of mathematics to design hospitals, schools, stadiums and all other construction projects (*Figure 1.1*). Once BAC Craftworkers are on the job, they use mathematics (or math) in all areas of the trowel trades (e.g., brick, cement, marble, plaster, pointing-cleaning-caulking, stone, tile, terrazzo, etc.).

HOW OLD IS MATH?

How old is the field of mathematics? The answer is, *many thousands of years*. Over the course of history, every culture has developed some form of mathematics. In some cases, mathematics has spread from one culture to another. In today's world, there is basically one international form of mathematics.

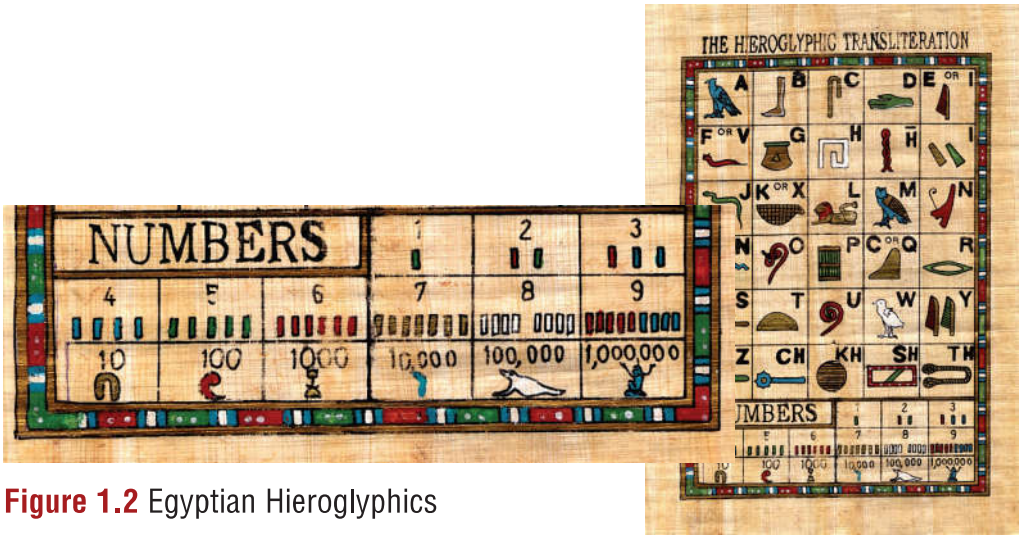

Figure 1.2 Egyptian Hieroglyphics

DEVELOPMENT OF MATHEMATICS

Figure 1.3 Egyptian Pyramids

Although mathematics has roots in ancient Egypt and Babylonia, it also grew rapidly in ancient Greece. Ancient Egyptians used a system of hieroglyphics – a form of writing in which pictures or symbols are used to represent objects, ideas, or sounds. *Figure 1.2* shows a Papyrus of hieroglyphic letters and numbers. The portion showing the numbers has been enlarged.

Egyptian scientists were generally most interested in observing nature and practical engineering. They were very good at both of these things. The pyramids (*Figure 1.3*) and temples, for example, show good knowledge of **geometry** and engineering. Egyptian engineers used the **Pythagorean Theorem** thousands of years before the Greek mathematician and philosopher Pythagoras (*Figure 1.4*) was born.

The ancient Greeks also had a strong interest in mathematics. One of the most famous Greek mathematicians was Pythagoras. The Pythagorean Theorem (which we will cover in this manual) is named after Pythagoras, who by tradition is credited with its discovery (although it is often argued that knowledge of the theory predates him).

Mathematics written in ancient Greek was translated into Arabic. About the same time, some mathematics of India was also translated into Arabic. Later, some of this mathematics was translated into Latin and became the mathematics of Western Europe. Over a period of several hundred years, it became the mathematics of the world.

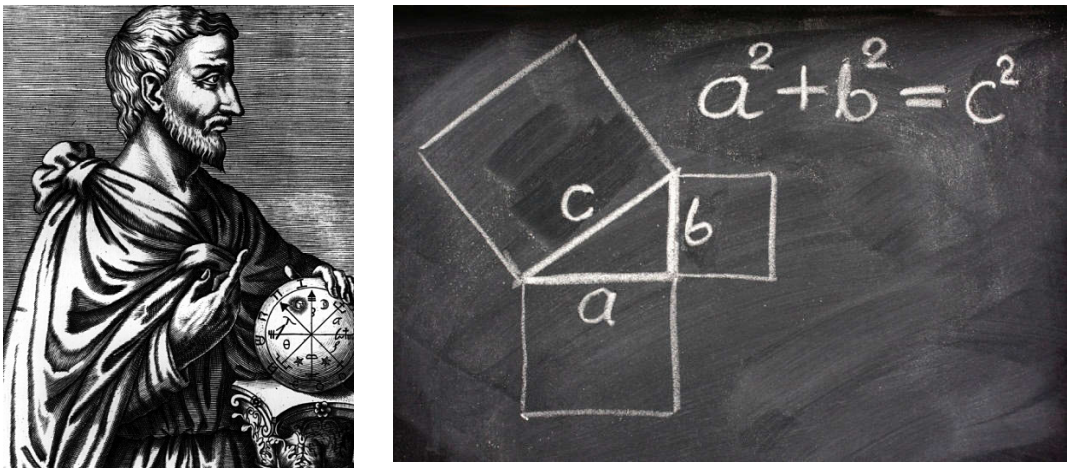

Figure 1.4 Pythagoras and Pythagorean Theorem (3-4-5 Right Triangle)

DEVELOPMENT OF MATHEMATICS TOOLS

As the field of mathematics evolved, mathematicians searched for tools to help them perform calculations. Although there were many simple and more complex tools that helped with mathematical calculations, we will look at the **abacus**, **slide rule**, calculator, and computer.

Figure 1.5 Old Abacus

ABACUS

An abacus (*Figure 1.5*), also called a *counting frame*, is a calculating tool used primarily in parts of Asia for performing arithmetic processes. The ancient abacus was first created in about 500 B.C. Today, abacuses are often constructed as a bamboo frame with beads sliding on wires, but originally they were beans or stones moved in grooves in sand or on tablets of wood, stone, or metal. The abacus was in use centuries before the adoption of the written, modern numeral system and is still widely used by merchants, traders, and clerks in Asia, Africa, and elsewhere.

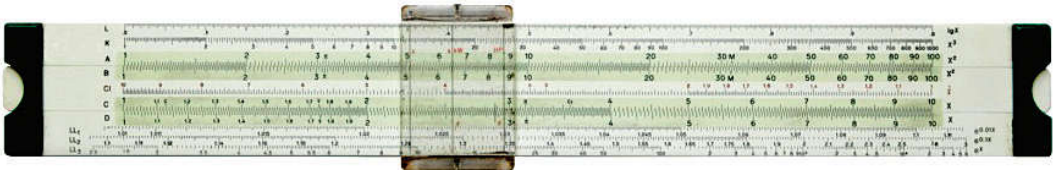

Figure 1.6 Slide Rule

SLIDE RULE

The slide rule (*Figure 1.6*), also known as a *slipstick*, is a mechanical calculator. The slide rule was first developed in the 1600s. It is used primarily for **multiplication**, **division**, and "scientific" functions, such as roots, logarithms, and **trigonometry**. A slipstick does not generally perform **addition** or **subtraction**.

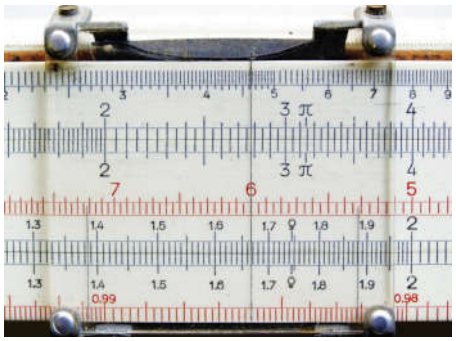

Figure 1.7 Close-Up of Slide Rule Scales

Slide rules come in a diverse range of styles. They generally appear in a **linear** or circular form with a standardized set of markings or scales (*Figure 1.7*) essential to performing mathematical computations. The use of slide rules continued to grow through the 1950s and 1960s, even as digital computing devices (i.e., computers and calculators) were being gradually introduced.

THE SCIENTIFIC CALCULATOR

Around 1974, the electronic **scientific calculator** made the slide rule largely obsolete and most suppliers exited the business.

A scientific calculator (*Figure 1.8*) is a type of electronic calculator, usually but not always handheld, designed to calculate problems in science, engineering, and mathematics. It has almost completely replaced slide rules in nearly all traditional applications and is widely used in both educational and professional settings. You will learn to use the scientific calculator in this manual.

Figure 1.8 Scientific Calculator

THE CONSTRUCTION CALCULATOR

Many BAC Craftworkers carry a **construction calculator** (*Figure 1.9*). These calculators typically allow you to easily determine precise **angle** measurements and solve complex design and construction-math problems (including trigonometric functions, 3-4-5 **right triangles**, and estimating materials and costs). You will learn to use a construction calculator in this manual.

Figure 1.9 Construction Calculator

COMPUTERS

The most advanced tool for performing mathematical calculations is the computer (*Figure 1.10*). Initially used by architects and engineers in the design of structures, computers are now found on most jobsites being used by Superintendents, General Foremen, and Foremen.

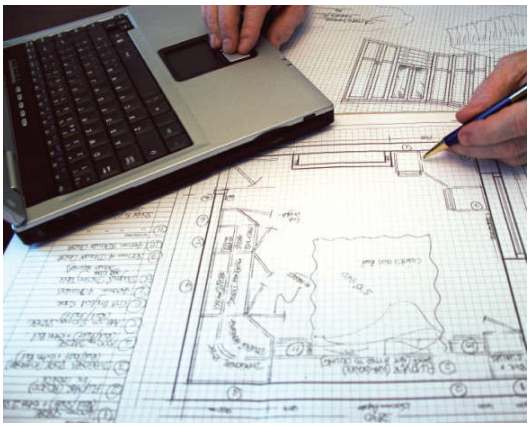

Figure 1.10 Laptop Computer

Now we know that mathematics has been around for thousands of years and that there are various tools we can use to assist with performing basic and advanced mathematical calculations. It is now time to look at the different numbering systems that a BAC Craftworker might use on a jobsite.

OBJECTIVE 2

SYSTEMS OF MEASUREMENT

BAC Craftworkers will use two basic systems of measurement:

1. United States Customary System
2. Metric System

UNITED STATES CUSTOMARY SYSTEM

The **United States Customary System of units of measurement** (also called the English, Imperial, American, or "Inch" system) is the primary and most commonly used system of measurement in the United States. The United States is one of only several countries still using this system of measurement. The rest of the countries of the world use the **metric system**.

In 1988, the United States government designated the metric system of measurement as "the preferred system of weights and measures for U.S. trade and commerce." The legislation stated that the federal government has a responsibility to assist industry, especially small business, as it voluntarily converts to the metric system of measurement. This process of legislation and conversion is known as *metrication*, and in the U.S. it is most evident in labeling requirements on food products, where metric units are almost always presented alongside customary units.

However, metrication in the United States has been less forcefully imposed than in other countries and has encountered more resistance from industrial and consumer market forces. For this reason, customary units are still widely used on consumer products and in industrial manufacturing. Only in military, medical, and scientific contexts are metric units generally the norm.

United States Customary System of measurement includes standards for many types of measures, including:

- Length (inch, foot, yard, mile)
- **Volume** (**cubic inch**, **cubic foot**, **cubic yard**)
- Mass or weight (ounce, pound, ton)
- Temperature (Fahrenheit)

Much of the content of this manual will be based on the United States Customary System of measurement.

METRIC SYSTEM

The metric system is an international decimal–based system of measurement founded by France in 1791. It is the common system of measuring units used by most of the world.

Since the 1960s, the International System of Units, or SI ("Système International d'Unités" in French, hence "SI"), has been the internationally recognized standard metric system. Metric units are widely used around the world for personal, commercial, and scientific purposes. A standard set of prefixes in powers of ten may be used to derive larger and smaller units from the base units.

The metric, or SI, system includes standards for many types of measures, including:

- Length (**millimeter**, **centimeter**, **meter**, **kilometer**)
- Volume (milliliter, cubic centimeter, **liter**, cubic meter)
- Mass or weight (milligram, **gram**, kilogram, metric ton)
- Temperature (centigrade)

On some of your jobs, the drawings will be in metric. When this occurs, the Craftworker may need to "convert" from metric to American measurements. Some calculators will do these conversions automatically. If one of these calculators is not available, then it may be necessary to use a conversion table like the one shown in *Table 1.1*.

American and Metric Length Conversions		
Unit	**Divisions**	**SI Equivalent**
1 inch (in)		25.4 mm
1 foot (ft)	12 in	0.3048 m
1 yard (yd)	3 ft	0.9144 m
1 mile (mi)	5,280 ft or 1,760 yd	1.609344 km

Table 1.1 American and Metric Length Conversions

In the *Advanced Mathematics* course, there is a unit focusing on the fundamentals of the metric system.

Fun with Numbers – Roman Numerals

Did you ever wonder what those "numbers" are that follow the Super Bowl title each year? They are Roman numerals. Roman numerals were used in Europe until the 18th century. Generally, the only place we see them today is as an alternative way of expressing a given number such as year, number on a building, chapter in a book, or a Super Bowl number. Here are the basic Roman numerals:

One	I	Eleven	XI	Thirty	XXX
Two	II	Twelve	XII	Forty	XL
Three	III	Thirteen	XIII	Fifty	L
Four	IV	Fourteen	XIV	Sixty	LX
Five	V	Fifteen	XV	Seventy	LXX
Six	VI	Sixteen	XVI	Eighty	LXXX
Seven	VII	Seventeen	XVII	Ninety	XC
Eight	VIII	Eighteen	XVIII	One hundred	C
Nine	IX	Nineteen	XIX	Five hundred	D
Ten	X	Twenty	XX	One thousand	M

This numbering system does look odd. Just imagine that you are a construction worker in ancient Rome. If you were asked to add two Roman values, such as CXXII + LXI, you would probably convert these two values to regular numbers, add them (122 + 61 = 183), and finally convert back to Roman numerals: CLXXXIII. This is because it is relatively easy to convert, and we know how to add ordinary decimal numbers.

The Romans could not do this! They needed a method of manipulating the Roman symbols directly to achieve the addition.

Fortunately for Roman engineers and accountants, there is a procedure for adding Roman numerals directly that is "supposedly" quite easy. Check out this example: CCCLXIX + DCCCXLV.

1.	Substitute for any subtractives to obtain:	CCCLXVIIII + DCCCXXXXV
2.	Catenate to obtain:	CCCLXVIIIIDCCCXXXXV
3.	Sort to obtain:	DCCCCCCLXXXXXVVIIII
4.	Combine groups to obtain:	DCCCCCCLXXXXXXIIII
		DCCCCCCLLXIIII
		DCCCCCCCXIIII
		DDCCXIIII
		MCCXIIII
5.	Substitute any subtractives to obtain:	MCCXIV

You can verify that this is indeed the correct answer by converting the values to regular notation: 369 + 845 = 1214.

I bet you're glad that you do not use Roman Numerals as a BAC Craftworker!!

OBJECTIVE 3

SELECT A CALCULATOR

Most BAC Foremen, General Foremen, and Superintendents will have access to some type of handheld calculator. In order to complete this mathematics course, you will need a calculator. So, how do you decide what type of calculator to purchase?

There are many different types of calculators. For instance, on the Internet, you can find a variety of free general calculators (e.g., basic math, **square root**, **percents**, etc.) and specialized calculators (e.g., financial, health, medical, etc.). There are also many different types of handheld calculators. In terms of handheld calculators for BAC Craftworkers, there are three basic options: the Basic Math Calculator, Scientific Calculator, and Construction Calculator.

> **Note:** Before purchasing a calculator, be sure to talk with your instructor to determine which calculator is recommended, not only for this mathematics course but for other courses as well.

BASIC MATH CALCULATOR

The basic math calculator (*Figure 1.11*) has the following characteristics:

- Very inexpensive.
- Very easy to use.
- Typically can add, subtract, multiply, divide, determine percentages and square roots, and has some basic memory functions.
- Can be used for the majority of the problems in this manual (although you will find the scientific and construction calculators can be used for all of the problems and are more useful on a jobsite).

Figure 1.11 Basic Calculator

SCIENTIFIC CALCULATOR

The scientific calculator (*Figure 1.12*) has the following characteristics:

- Relatively inexpensive.
- Relatively easy to use.
- Can perform all of the general functions needed by a BAC Craftworker (including all of the trigonometry functions).
- Can be used for all of the problems in this manual.

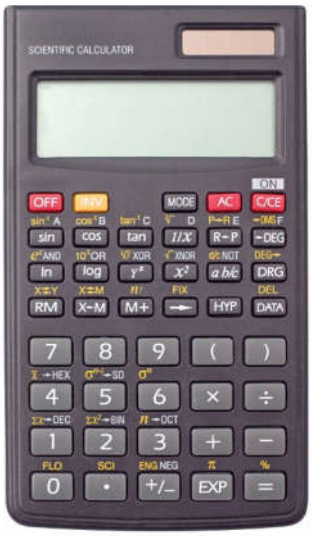

Figure 1.12 Scientific Calculator

CONSTRUCTION CALCULATOR

The construction calculator (*Figure 1.13*) has the following characteristics:

- Typically more expensive than a general scientific calculator.
- Designed for the construction professional.
- Typically can perform all of the basic functions, as well as construction related functions (e.g., circular calculations, 3-4-5 right triangles, stair layouts, weight per volume calculations, dimensional math conversions, etc.).
- Some models also include the full trigonometric functions.
- Can be used for all of the problems in this manual.

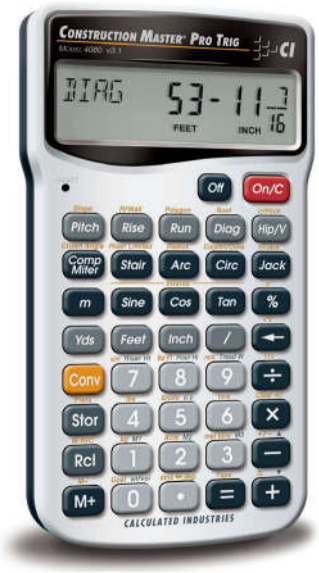

Figure 1.13 Construction Calculator

Note: Always keep the manufacturer's instruction guide that comes with your calculator, as you will need to refer to the instructions when you work on different types of problems.

CALCULATORS IN THIS COURSE

In this manual, we will show you how to use your calculator to solve math problems. In order to do this, we will show you the sequence to press specific keys (keystrokes) on the calculator. Note the following important points:

- We will use both a scientific calculator and a construction calculator to show you how to solve problems.
- We will show the keystrokes on the calculator (*Figure 1.14*). Given that your calculator will be similar to, but likely different from the calculators we are using, you will need to refer to the manufacturer's instruction guide for specific keystrokes using your calculator.

Figure 1.14 Sample Calculator Keystrokes

Note: In this manual, we recommend NOT using your calculator until after completion of Unit 3. This will help you learn the needed basic math skills before depending on the calculator.

OBJECTIVE 4

HOW TO USE THIS STUDENT MANUAL

This manual will focus on the mathematics needed by BAC Craftworkers. You may have little experience with math, or you may be very familiar with math. You may like math, or you may dislike math. Regardless, understanding and applying basic mathematics is an important part of being a skilled Craftworker.

We will begin with the very basics of adding, subtracting, multiplying, and dividing whole numbers (Unit 2), common fractions (Unit 3), and decimal fractions (Unit 4).

In Unit 5, we will learn about linear, area, circular, and volume measurement.

The key to learning mathematics is practice, practice, and more practice. In this manual, we will show you many examples and provide you with many opportunities to practice and learn math in a BAC Craftworker context.

Learning the content of this manual will make you a better and more successful Craftworker.

UNIT 2

WHOLE NUMBERS

OBJECTIVES

After completion of this unit, you should be able to solve mathematics problems involving the addition, subtraction, multiplication, and division of whole numbers. This knowledge will be evidenced by correctly completing the Assignment Sheets and by scoring a minimum of 70% on the Unit Test.

Specifically, you should be able to:

1. Add whole numbers.
2. Subtract whole numbers.
3. Multiply whole numbers.
4. Divide whole numbers.
5. Perform combined operations with whole numbers.

Each of these objectives is covered in the pages that follow.

OBJECTIVE 1

ADDITION OF WHOLE NUMBERS

Note: You should be able to perform operations with whole numbers without the aid of a calculator. It is expected that the Assignment Sheets and test for this unit will be completed without the aid of a calculator.

Numbers are words or figures that indicate a quantity (e.g., 10 pounds, 47 bricks, or 65 miles). Whole numbers (*Figure 2.1*) refer to complete numbers with no fractional parts of the number left over. For example, 50 is a whole number.

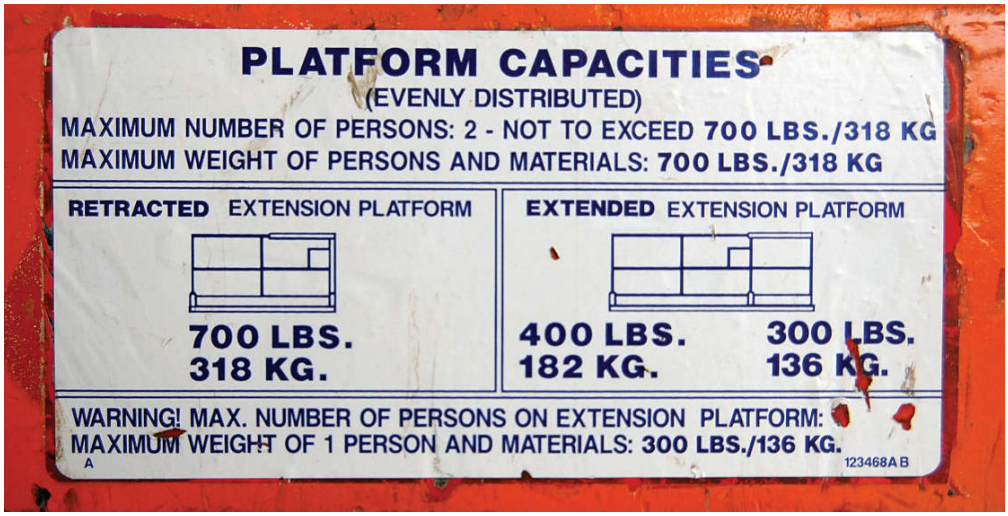

Figure 2.1 Whole Numbers on a Platform Lift

Most of the numbers used by BAC Craftworkers are whole numbers and fractions of whole numbers (e.g., 6 ¾ inches, 15.2 tons). Note that fractions will be discussed in other units of this manual.

DIGITS AND PLACES

In the U.S. customary numbering system, there are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We can combine these ten digits to express any number. When we write these numbers, each digit will have a "place" that indicates a specific value. To show the values of these places, we can write numbers in columns. Each of these columns or "places" indicates a specific value, as shown in *Table 2.1*.

Place Values for Whole Numbers						
Millions	Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Units (or Ones)
3	5	8	4	9	6	2

Table 2.1 Place Values for Whole Numbers

The number shown in *Table 2.1* is read 3,584,962 or three million, five hundred eighty-four thousand, nine hundred sixty-two.

GROUPING NUMBERS

When writing whole numbers, commas may be inserted to group the numbers in threes, beginning from the right end of the number. As seen in the following examples, this makes the numbers easier to read.

8,450 pounds

5,280 feet

14,750 bricks

1,500 pieces of tile

You may also see groupings of three using only spaces to separate the groups. This is shown in the following examples.

8 450 pounds

5 280 feet

14 750 bricks

1 500 pieces of tile

In some cases, you will see numbers expressed with no commas or spaces, as in *8450 pounds.*

MAKING SUMS

Addition is the process of combining two or more numbers into one number. The number we get when we add two or more numbers is called the ***sum***. The **plus sign** (+) is used to indicate that two or more numbers are to be added. The **equals sign** (=) is used to indicate the sum of the numbers being added.

> **Note:** You can only add numbers with the same units of measurement. For example, you can add inches and inches. You cannot add inches and feet! This is where the expression "You can't add apples and oranges" comes from.

ADDING SINGLE-DIGIT NUMBERS

Adding single–digit whole numbers is easy. If you have 5 bricks and pick up 3 more bricks, then you would have 5 + 3 = 8 bricks (*Figure 2.2*). This addition problem could also be written in column format as shown in the following example.

Figure 2.2 Adding Bricks

5 bricks
+ 3 bricks
8 bricks

ADDING MULTI-DIGIT NUMBERS

When adding whole numbers of 10 or greater (more than 1 digit), it is often easier to place the numbers in columns. Review the following examples.

Example: Add 214 pounds + 69 pounds.

Step 1: Starting on the right and moving left, line the numbers up in columns. Be sure to line up similar units in the same columns: In this example, the 4 and the 9 are single units or "ones."

214
+ 69

Step 2: Add the numbers in the units column. In this example, we add 4 and 9. The sum is 13. As we only have space for the 3, we "carry" the 1 to the top of the next column. We are really carrying a "ten" to the tens column.

1
214
+ 69
3

Step 3: Add the numbers in the tens column. We have 1 + 6 + the 1 we carried for a total of 8. There is nothing to carry to the hundreds column.

```
  1
 214
+ 69
 283
```

Answer: 283 pounds

Example: Add 7,765 + 2,983 + 156 + 370.

Step 1: Starting on the right and moving left, line the numbers up in columns.

```
 7765
 2983
  156
+ 370
```

Step 2: Add the numbers in the units column. In this example, we add 5 + 3 + 6 + 0 = 14. We write the 4, and carry the 1 to the top of the next column.

```
   1
 7765
 2983
  156
+ 370
    4
```

Step 3: Add the numbers in the tens column. The sum is 27. Write the 7, and carry the 2.

```
  21
 7765
 2983
  156
+ 370
   74
```

Step 4: Add the numbers in the hundreds column. The sum is 22. Write the 2, and carry the 2.

$$\begin{array}{r} 221 \\ 7765 \\ 2983 \\ 156 \\ +\ 370 \\ \hline 274 \end{array}$$

Step 5: Add the numbers in the thousands column. The sum is 11. Write the 1, and carry the 1.

$$\begin{array}{r} 1221 \\ 7765 \\ 2983 \\ 156 \\ +\ 370 \\ \hline 1274 \end{array}$$

Step 6: Add the numbers in the ten-thousands column. We only have the 1 we carried, so the sum is 1.

$$\begin{array}{r} 1221 \\ 7765 \\ 2983 \\ 156 \\ +\ 370 \\ \hline 11274 \end{array}$$

Answer: 11,274

Example: Add 13,980 + 7,638 + 5,649 + 16,000. Work out this problem on another piece of paper to make sure that you get the correct answer.

$$\begin{array}{r} 13{,}980 \\ 7{,}638 \\ 5{,}649 \\ +\ 16{,}000 \\ \hline 43{,}267 \end{array}$$

Answer: 43,267

UNIT 2

CHECKING YOUR ADDITION

The best method to check your answer in an addition problem is to change the order of the numbers, and then add them again. The order in which you add the numbers will not change the answer.

> **Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 2

SUBTRACTION OF WHOLE NUMBERS

Subtraction is the process of finding the difference between two numbers. It is the inverse, or opposite, of addition. This means that if we start with any number, add any number, and then subtract the same number we added, we return to the number we started with.

For example, suppose that you want to find the difference between 45 feet and 18 feet. In this case, you would subtract 18 feet from 45 feet. This is written as **45 – 18 = _____.**

MINUENDS AND SUBTRAHENDS

The larger of the two numbers is the **minuend** (the number on top in the following example). The smaller number is the **subtrahend** (the number on the bottom).

45	**minuend**
– 18	**subtrahend**

The **minus sign** (–) is used to indicate that one number is to be subtracted from another number. The equals sign (=) is used to indicate the **difference** (or answer) of the numbers being subtracted.

> **Note:** You can only subtract numbers with the same units of measurement. For example, you can subtract feet from feet. You cannot subtract feet from inches!

SUBTRACTING SINGLE-DIGIT NUMBERS

Subtracting single–digit whole numbers is easy. If you have 7 tools, and someone borrows 3 tools, then you would have 7 – 3 = 4 tools remaining (*Figure 2.3*). As shown in the following example, this subtraction problem could also be written in a column format.

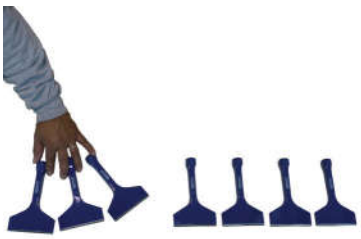

Figure 2.3 Removing (or Subtracting) Tools

7 tools
– 3 tools
4 tools

Note: If you subtract one number that is larger than the number it is being subtracted from, this will result in a negative number (i.e., a number less than zero). For example, in this problem, 3 – 7 = - 4 (or a "negative" 4). In this manual, we will only work with positive numbers.

SUBTRACTING MULTI-DIGIT NUMBERS

When subtracting numbers with more than 1 digit (e.g., 10), it is often easier to place the numbers in columns. Review the following examples.

Example: Subtract 147 feet from 868 feet.

Step 1: Line up the numbers in columns with the larger number on top (the minuend).

$$\begin{array}{r} 868 \\ -\ 147 \\ \hline \end{array}$$

Step 2: Subtract the numbers in the units column. In this example, the difference is 8 – 7 = 1.

$$\begin{array}{r} 868 \\ -\ 147 \\ \hline 1 \end{array}$$

Step 3: Subtract the numbers in the tens column. Here, the difference is 6 – 4 = 2.

$$\begin{array}{r} 868 \\ -\ 147 \\ \hline 21 \end{array}$$

Step 4: Subtract the numbers in the hundreds column. The difference is 8 – 1 = 7.

$$\begin{array}{r} 868 \\ -\ 147 \\ \hline 721 \end{array}$$

Answer: 721 feet

BORROWING WHEN SUBTRACTING

Subtraction of whole numbers becomes a little more complicated when a digit in the subtrahend is greater than the digit in the minuend. When this happens, you will need to "borrow" from the column to the left. So, if you are in the units column, then you borrow from the tens column. If you are in the tens column, then you borrow from the hundreds column.

Example: Subtract 68 degrees from 92 degrees. Use the following steps.

Step 1: Since 8 is greater than 2, we will need to borrow 1 of the 10s. There are nine 10s, so when we move one of these 10s, we are left with 8. Since we are bringing a "10" to the units column, we will now have 10 + 2 = 12 in the units column. The difference can now be determined as 12 – 8 = 4.

$$\begin{array}{r} {\scriptstyle 1}2 \\ \cancel{9}2 \\ -\ 68 \\ \hline 4 \end{array}$$

Step 2: In the next column, we now have 8 – 6 = 2. Remember that we borrowed one of the 10s in this column, leaving eight 10s.

$$\begin{array}{r} 8{\scriptstyle 1}2 \\ \cancel{9}2 \\ -\ 68 \\ \hline 24 \end{array}$$

Answer: 24 degrees

CHECKING YOUR SUBTRACTION

To check the answer in a subtraction problem, simply add the answer to the subtrahend. This should give you the minuend. From the previous example, we would have 24 + 68 = 92. If the answer is the same as the minuend, then the subtraction has been completed correctly.

Example: Subtract 1,268 from 2,175, and then check your answer.

Step 1: Since 8 is greater than 5, we borrow 1 of the 10s from the next column. This gives us 15 – 8 = 7.

$$\begin{array}{r} {\scriptstyle 1}5 \\ 2{,}17\cancel{5} \\ -\ 1{,}268 \\ \hline 7 \end{array}$$

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Step 2: In the second (or tens) column, we now have 6 – 6 = 0.

$$\begin{array}{r} 6^{1}5 \\ 2{,}1\cancel{75} \\ -\ 1{,}268 \\ \hline 07 \end{array}$$

Step 3: In the third (or hundreds) column, the number 2 is greater than 1, so we borrow 1 of the 1000s from the fourth column. This gives us 11 – 2 = 9.

$$\begin{array}{r} 1^{1}6^{1}5 \\ 2{,}\cancel{175} \\ -\ 1{,}268 \\ \hline 907 \end{array}$$

Step 4: In the fourth (or thousands) column, we now have 1 – 1 = 0.

$$\begin{array}{r} 1\,{}^{1}1^{1}6^{1}5 \\ \cancel{2}{,}\cancel{175} \\ -\ 1{,}268 \\ \hline 0\ 907 \end{array}$$

Answer: 907

To check the answer, add the answer to the subtrahend as shown:

907 + 1,268 = 2,175

When taking tests, keep this answer-checking process in mind, as you should be able to verify each answer is correct.

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 3

MULTIPLICATION OF WHOLE NUMBERS

Multiplication is a fast method of adding the same number more than once. For example, suppose you have 42 boxes with 6 modular rules in each box (*Figure 2.4*). To determine the total number of modular rules, you could add the number in each box (6) a total of 42 times. Performing this calculation, however, is not recommended, as it would take a lot of time and would likely result in one or more mistakes. Using multiplication is much quicker.

Figure 2.4 Box of Modular Rules

UNIT 2

MULTIPLICATION TERMS

The **multiplicand** is the number to be multiplied (6 in our example). The other number (42 in our example) is the **multiplier**.

The **times sign** (×) is used to indicate that one number is to be multiplied by another number. The **equals sign** (=) is used to indicate the **product** (or the answer) of the numbers being multiplied.

MULTIPLYING SINGLE-DIGIT NUMBERS

Multiplying single–digit whole numbers is fairly easy. If you have 3 tubes of caulk in a box and 4 boxes, then you would have 3 + 3 + 3 + 3 = 12 tubes of caulk. To perform this calculation using multiplication, do the following:

$$3 \times 4 = 12$$

This multiplication problem could also be written in a column format as shown in the following example.

3	**multiplicand**
× 4	**multiplier**
12	**product**

Note: Keep in mind the *commutative property of multiplication*: When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands.
For example: 3 × 4 = 4 × 3.

MULTIPLICATION TABLE

There are 10 basic numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). **Note that 0 times any number is still 0**. To help learn to multiply combinations of the other numbers, it is important to learn the multiplication table (*Table 2.2*).

> **Note:** You will not be able to use the multiplication table or a calculator on the Unit Test, so it is essential to memorize the multiplication facts from this table.

Multiplication Table									
	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Table 2.2 Multiplication Table

Here are some examples from the multiplication table (*Table 2.2*).

$3 \times 4 = 12$

$7 \times 9 = 63$

$6 \times 6 = 36$

MULTIPLYING MULTI-DIGIT NUMBERS

When multiplying numbers greater than 9, it is easier to place the numbers in columns. **Note in multiplication that each number in the multiplicand is multiplied by each number in the multiplier**. Review the following examples.

Example: Multiply 54 × 68.

Step 1: Begin with the first column on the right, giving us 8 × 4 = 32. Record the 2 in the first column, and carry the 3.

$$\begin{array}{r} {\scriptstyle 3} \\ 54 \\ \times\ 68 \\ \hline 2 \end{array}$$

Step 2: Still in the first column, multiply 8 × 5 = 40. Adding the 3 that we carried from the first column, we have 40 + 3 = 43.
Thus, 8 × 54 = 432.

$$\begin{array}{r} {\scriptstyle 3} \\ 54 \\ \times\ 68 \\ \hline 432 \end{array}$$

Step 3: We are now working with 6 in the second, or 10s, column. Because we are multiplying by 10s and need to keep columns aligned, insert 0 in the first column of the second row. Multiply 6 × 4 = 24. Record 4 in second column, and carry the 2.

$$\begin{array}{r} {\scriptstyle 2} \\ {\scriptstyle 3} \\ 54 \\ \times\ 68 \\ \hline 432 \\ +\ 40 \end{array}$$

Step 4: Now, multiply 6 × 5 = 30. Adding the 2 that we carried, we have 30 + 2 = 32. Record the 32 in the third and fourth columns. Note that 60 (we have six 10s or 60) times 54 is 3,240.

$$\begin{array}{r} {\scriptstyle 2} \\ {\scriptstyle 3} \\ 54 \\ \times\ 68 \\ \hline 432 \\ +\ 3240 \end{array}$$

Step 5: Add the numbers in the four columns of the bottom two rows.

Answer: 3,672

UNIT 2

Example: Multiply 58 × 29.

Step 1: Begin with the first column on the right, giving us 9 × 8 = 72. Record the 2 in the first column, and carry the 7.

$$\begin{array}{r} {\scriptstyle 7} \\ 58 \\ \times\ 29 \\ \hline 2 \end{array}$$

Step 2: Still in the first column, multiply 9 × 5 = 45. Adding the 7 that we carried from the first column, we have 45 + 7 = 52.
Thus, 9 × 58 = 522.

$$\begin{array}{r} {\scriptstyle 7} \\ 58 \\ \times\ 29 \\ \hline 522 \end{array}$$

Step 3: We are now working with the 2 in the second column. Insert a zero in the first column of the second row. Now, multiply 2 × 8 = 16. Record the 6 in the second column, and carry the 1.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 7} \\ 58 \\ \times\ 29 \\ \hline 522 \\ +\ 60 \end{array}$$

Step 4: Now, multiply 2 × 5 = 10. Adding the 1 that we carried, we have 10 + 1 = 11. Record the 11 in the third and fourth columns.

$$\begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 7} \\ 58 \\ \times\ 29 \\ \hline 522 \\ +\ 1160 \end{array}$$

Step 5: Add the numbers in the four columns of the bottom two rows.

Answer: 1,682

Example: Multiply 4,762 × 85.

Step 1: Multiply 5 × 4,762. Begin with 5 × 2 = 10. Record the 0, and carry the 1.

$$\begin{array}{r} 1 \\ 4762 \\ \times\ 85 \\ \hline 0 \end{array}$$

Step 2: Multiply 5 × 6 = 30. Add the 1, and the sum is 31. Record the 1, and carry the 3.

$$\begin{array}{r} 31 \\ 4762 \\ \times\ 85 \\ \hline 10 \end{array}$$

Step 3: Multiply 5 × 7 = 35. Add the 3, and the sum is 38. Record the 8, and carry the 3.

$$\begin{array}{r} 331 \\ 4762 \\ \times\ 85 \\ \hline 810 \end{array}$$

Step 4: Multiply 5 × 4 = 20. Add the 3, and the sum is 23. Record the 23 in the fourth and fifth columns.

$$\begin{array}{r} 331 \\ 4762 \\ \times\ 85 \\ \hline 23810 \end{array}$$

So, 5 × 4,762 = 23,810.

Step 5: Multiply 8 × 4,762. Start by inserting a zero in the first column of the second row. Begin with 8 × 2 = 16. Record the 6 in the second column, and carry the 1.

$$\begin{array}{r} 1 \\ 4762 \\ \times\ 85 \\ \hline 23810 \\ +\ 60 \end{array}$$

UNIT 2

Step 6: Multiply 8 × 6 = 48. Add the 1, and the sum is 49. Record the 9, and carry the 4.

```
   41
 4762
×  85
23810
+ 960
```

Step 7: Multiply 8 × 7 = 56. Add the 4, and the sum is 60. Record the 0, and carry the 6.

```
  641
 4762
×  85
23810
+ 0960
```

Step 8: Multiply 8 × 4 = 32. Add the 6, and the sum is 38. Record the 38 in the fifth and sixth columns.

```
  641
 4762
×  85
23810
+ 380960
```

So, 80 × 4,762 = 380,960.

Step 9: Add the numbers in the six columns of the bottom two rows.

Answer: 404,770

MULTIPLICATION: EASY AND FAST

Although the examples may seem a little complicated at first, after some practice, multiplication problems like this one are fairly easy. Remember that the other option to solve this problem is to write 4,762 in a column 85 times, and then add the numbers. Multiplication is much faster, plus you are less likely to make mistakes.

MULTIPLYING BY ZERO

It was previously mentioned that multiplying any number times 0 results in 0. Here is an example where 0 is one of the numbers. Note how this is handled in the following example.

Example: Multiply 309 × 25.

Step 1: Multiply 5 × 309. Begin with 5 × 9 = 45. Record the 5, and carry the 4.

$$\begin{array}{r} {\scriptstyle 4} \\ 309 \\ \times\ 25 \\ \hline 5 \end{array}$$

Step 2: Multiply 5 × 0 = 0. Add the 4 carried from the first column, and the sum is 4.

$$\begin{array}{r} {\scriptstyle 4} \\ 309 \\ \times\ 25 \\ \hline 45 \end{array}$$

Step 3: Multiply 5 × 3 = 15. Record 15 in the third and fourth columns.

$$\begin{array}{r} {\scriptstyle 4} \\ 309 \\ \times\ 25 \\ \hline 1545 \end{array}$$

Step 4: Insert a 0 in the first column of the second row, and then multiply 2 × 309. Verify that the product is 618 as shown.

$$\begin{array}{r} {\scriptstyle 4} \\ 309 \\ \times\ 25 \\ 1545 \\ +\ 6180 \\ \hline \end{array}$$

Step 5: Add the numbers in the four columns of the bottom two rows.

Answer: 7,725

UNIT 2

CHECKING YOUR MULTIPLICATION

The best method to check your answer in a multiplication problem is to reverse the order of the numbers, and then multiply them again. The order in which you multiply the numbers will not change the answer. Keep this process in mind when taking the Unit Test, as you should be able to verify that each answer is correct.

Example: Multiply 42 × 93.

Step 1: Complete the multiplication to show that the product, or answer, is 3,906.

Step 2: To check your answer, multiply 93 × 42 to determine that the answer is still 3,906.

Fun with Numbers – Multiplication by 11

Before there were calculators, mathematicians looked for shortcuts with solving problems. We will now look at some "fun" ways to play with numbers. Here are some fun math facts to keep in mind when multiplying by 11.

When multiplying two digit numbers: Add the two digits, and keep the answer in between the two digits. If the addition comes to more than 10, then carry over and add to the left-hand digit.

68 × 11

= 6 (6 + 8) 8

= 6 (14) 8

= 6 + 1(4) 8

= 7 4 8

= 748

What happens if there are more than two digits? Keep the extreme digits on their respective extreme sides, then pair off digits starting from the left, and add the pairs from the right, keeping the answer in their respective positions, carrying over if required.

7964 x 11

= 7 (7 + 9) (9 + 6) (6 + 4) 4

= 7 (16) (15) (10) 4

= 7 (16) (15 + 1) (0) 4

= 7 (16) (16) 0 4

= 7 (16 + 1) (6) 0 4

= 7 (17) 6 0 4

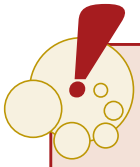
= 7 + 1 (7) 6 0 4

= 8 7 6 0 4

= 87604

This process is quite amazing and fun, especially for large numbers!

UNIT 2



Fun with Numbers – Multiplying in Your Head

Here is a quick way to multiply any two numbers between 11 and 20 in your head.

Take 15 × 13 for example.

- Always place the larger number of the two on top in your mind.
- Then, draw the shape of Africa mentally, so it covers the 15 and the 3 from the 13 below. Those covered numbers are all you need.
- First, add 15 + 3 = 18.
- Add a zero behind it (really multiplying by 10) to get 180.
- Multiply the covered lower 3 × the single digit above it (in this example the 5), which is (3 × 5 = 15).
- Add 180 + 15 = 195.

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

OBJECTIVE 4

DIVISION OF WHOLE NUMBERS

Division is a process of determining how many times one number is contained in another number. For example, suppose that you need 72 fasteners for an installation. The fasteners come in boxes of 8. How many boxes will you need?

The number to be divided (72 in our example) is known as the **dividend**. The number being divided into the dividend (8 in our example) is the **divisor**.

DIVISION SIGNS AND SYMBOLS

The **division sign** (÷) is used to indicate that one number is to be divided by another number. The equals sign (=) is used to indicate the **quotient** (or the answer) of the numbers being divided. In our example, the equation would be written as follows:

$$72 \div 8 = ?$$

The other common symbol for division is shown in red in the following equation:

$$8 \overline{)\,72}^{\,?}$$

In this division problem, we are asking the following question: How many times can I divide 8 into 72? Or, how many times will 8 "go into" 72? Going back to the multiplication table, we see that $9 \times 8 = 72$. So, we know that $72 \div 8 = 9$. We also know that $72 \div 9 = 8$.

DIVISION WITH REMAINDERS

In the previous example, the number 8 can be divided into 72 exactly 9 times. This means that if you had 72 fasteners, you could make 9 piles of 8 fasteners. If you had 75 fasteners and tried to make 9 piles, you could make 9 piles and would have 3 left over or remaining. The number "left over" after dividing two numbers is known as the **remainder**.

Consider the equation $49 \div 9$. According to the multiplication table, $9 \times 5 = 45$ and $9 \times 6 = 54$, so the solution is $49 \div 9 = 5$ with a remainder of 4.

Example: Divide 49 by 9.

Step 1: Begin by determining if the 9 will divide into the 4. Because 9 is larger than 4, we move on to determine if 9 will divide into 49. According to the multiplication table, $9 \times 5 = 45$. Record the 5 above the 9.

$$9\overline{)\,49}\quad \text{(5 above the 9)}$$

Step 2: Multiply the 5 in the quotient times the divisor (9). This is $5 \times 9 = 45$. Record the 45 under the 49.

$$\begin{array}{r} 5 \\ 9\overline{)\,49} \\ -\,45 \end{array}$$

Step 3: Subtract $49 - 45 = 4$. Since 9 will not divide into 4, 4 is the remainder.

$$\begin{array}{r} 5 \\ 9\overline{)\,49} \\ -\,45 \\ \hline 4 \end{array}$$

Answer: $49 \div 9 = 5$ with a remainder of 4

Example: Divide 386 by 27.

Step 1: There are two numbers in the divisor, so look at the first two numbers in the dividend. Think $38 \div 27$. By inspection, we see that 27 will divide into 38 one time. Record the 1 in the quotient.

$$27\overline{)\,386}\quad \text{(1 above the 8)}$$

Step 2: Multiply the 1 in the quotient times the divisor (27). This is $27 \times 1 = 27$. Record the 27 under the 38.

$$\begin{array}{r} 1 \\ 27\overline{)\,386} \\ -\,27 \end{array}$$

Step 3: Subtract $38 - 27 = 11$. Bring "down" the 6 to make this 116.

$$\begin{array}{r} 1 \\ 27\overline{)\,386} \\ -\,27 \\ \hline 116 \end{array}$$

Step 4: Now divide 27 into 116. This is where estimating quotients will be helpful. Since 27 is almost 30, and 30 × 4 = 120, the answer is probably close to 4. Estimate 4, and record this in the quotient.

$$\begin{array}{r} 14 \\ 27\overline{)\,386} \\ -\,27 \\ \hline 116 \end{array}$$

Step 5: Multiply the 4 in the quotient times the divisor (27): 27 × 4 = 108. Record the 108 under the 116.

$$\begin{array}{r} 14 \\ 27\overline{)\,386} \\ -\,27 \\ \hline 116 \\ -\,108 \\ \hline \end{array}$$

Step 6: Subtract 116 – 108 = 8. Since 8 is smaller than 27, 8 is the remainder.

$$\begin{array}{r} 14 \\ 27\overline{)\,386} \\ -\,27 \\ \hline 116 \\ -\,108 \\ \hline 8 \end{array}$$

Answer: 386 ÷ 27 = 14 with a remainder of 8

CHECKING DIVISION

To check that your answer is correct: Multiply the quotient by the divisor, and then add the remainder (if there is a remainder).

Examples: In a previous problem, we found that 72 ÷ 9 = 8. To check this answer, multiply the quotient (8) by the divisor (9), that is 9 × 8 = 72. Since 72 was our dividend, the answer is correct.

We also found that 49 ÷ 9 = 5 with a remainder of 4. Here is how to check this answer:

Step 1: Multiply the quotient by the divisor: 5 × 9 = 45
Step 2: Add the remainder: 45 + 4 = 49

Keep this process in mind when taking the Unit Test, as you should be able to verify that each answer is correct.

PRACTICE FOR SUCCESS

The key to developing the ability to divide whole numbers is practice. Here are some problems with their answers. Using another piece of paper, verify that the answers are correct.

216 ÷ 12 = 18

387 ÷ 23 = 16 with a remainder of 19

1,256 ÷ 83 = 15 with a remainder of 11

A number divided by 16 is 21 with a remainder of 7. What is the number? The answer is 343.

A number divided by 150 is 62 with a remainder of 54. What is the number? The answer is 9,354.

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

Fun with Numbers – Divisibility Rules

Rules, rules, rules— No one likes to follow rules, but sometimes there are good rules, like the ones that allow you to do less work! Well, these are exactly the kind of rules that you will find on this page.

There are many shortcuts or tricks that allow you to test whether a number or **dividend** is **divisible** by a given **divisor** with no remainder. These fun facts focus on the most frequently used rules, which involve dividing a number by 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11.

Divisibility by 2 rule: All even numbers can be divided by 2.

Divisibility by 3 rule: A number is divisible by 3 if the sum of its digits is divisible by 3.

Is 4,327,573,698 divisible by three? Let's apply the rule.

4 + 3 + 2 + 7 + 5 + 7 + 3 + 6 + 9 + 8 = 54 54 ÷ 3 = 18

Therefore, we know that the entire number is divisible by 3

Divisibility by 4 rule: A number is divisible by 4 if the last two digits of that number are divisible by 4.

Divisibility by 5 rule: A number is divisible by 5 if it ends in either 0 or 5.

Divisibility by 6 rule: A number is divisible by 6 if it is divisible by both two and three. So, we can deduce that all even numbers that are divisible by 3 are also divisible by 6.

Divisibility by 7 rule: To determine if a number is divisible by 7, take the last digit off the number, double it, and subtract the doubled number from the remaining number. If the result is evenly divisible by 7 (e.g., 14, 7, 0, -7, etc.), then the number is divisible by seven. This may need to be repeated several times.

Is 3,101 evenly divisible by 7?

310 - Take off the last digit of the number, which was 1.

<u>– 2</u> - Double the removed digit, and subtract it.

308 - Repeat the process by taking off the 8.

<u>– 16</u> - Double it to get 16, which is subtracted.

14 - The result is 14, which is a multiple of 7.

Divisibility by 8 rule: A number is divisible by 8 if the last three digits of that number are divisible by 8.

Divisibility by 9 rule: A number is divisible by 9 if the sum of the digits are divisible by 9.

Divisibility by 10 rule: A number is divisible by 10 if it ends in 0. So, we can say that all numbers ending in 0 are divisible by 10.

Divisibility by 11 rule: A number is divisible by 11 if the difference of the sums of alternating digits is divisible by 11 (this may be clearer with a few examples).

946 = (9 + 6) – 4 = 11 which is, of course, evenly divided by 11, so 946 passes this divisibility test

10,813 = (1 + 8 + 3) – (0 + 1) = 12 – 1 = 11 Yes, this satisfies the rule for 11.

25,784 = (2 + 7 + 4) – (5 + 8) = 13 – 13 = 0

Yes, this does indeed work. In case you found this last bit confusing, remember that any number evenly divides 0. Think about it, how many 11's are there in 0? None, right? Well that means that 11 divides zero, zero times!

119,777,658 = (1 + 9 + 7 + 6 + 8) – (1 + 7 + 7 + 5) = 31 – 20 = 11

OBJECTIVE 5

COMBINED OPERATIONS WITH WHOLE NUMBERS

The four basic functions (addition, subtraction, multiplication, and division) are used to solve many problems in the trowel trades. When more than one of the four basic functions appears in a problem, a specific sequence must be followed in order to correctly solve the problem. This is referred to as "mathematical order of operations."

MATHEMATICAL ORDER OF OPERATIONS

Here is the order in which the operations should be performed:

1. Simplify any expression inside a grouping symbol (e.g., parentheses and brackets).

   $5 + (6 \div 3) = 5 + 2 = 7$

   $8\,(15 - [2 \times 3]) = 8\,(15 - 6) = 8\,(9) = 8 \times 9 = 72$

   $(12 \times 4) \div (2\,[2 + 1]) = (48) \div (2 \times 3) = 48 \div 6 = 8$

2. Simplify expressions with exponents (exponents will be covered later in this manual).
3. Carry out multiplication or division from *left to right*.
4. Simplify addition or subtraction from *left to right*.

   $17 - 3 \times 4 = 17 - 12 = 5$

   $16 \div 8 + 4 = 2 + 4 = 6$

   $27 - 3 \times 5 + 6 \div 2 = 27 - 15 + 3 = 15$

Note: The abbreviation P.E.M.D.A.S, or "Please Excuse My Dear Aunt Sally," will help you to remember the order of operations. This means:

1st Parentheses

2nd Exponents

3rd Multiplication & Division

4th Addition & Subtraction

UNIT 2

PRACTICING ORDER OF OPERATIONS (PEMDAS)

Some additional examples will show how to perform combined operations with whole numbers.

Examples: Solve $12 \div (3 \times 2) + 5 - 2 \times 2$. See the problem expanded below showing each step.

$12 \div (3 \times 2) + 5 - 2 \times 2$	**original problem**
$12 \div 6 + 5 - 2 \times 2$	**performed operation in parentheses**
$2 + 5 - 4$	**performed division and multiplication**
$7 - 4 = 3$	

Solve $432 \div (9 + 7) + 14 \times 3$.

$432 \div (9 + 7) + 14 \times 3$	**original problem**
$432 \div 16 + 14 \times 3$	**performed operation in parentheses**
$27 + 42 = 69$	**divided, multiplied, and then added**

Solve $18 + 6 \times 5 - (150 \div 25) + 8$.

$18 + 6 \times 5 - (150 \div 25) + 8$	**original problem**
$18 + 30 - 6 + 8$	**performed division in parentheses**
$48 - 6 + 8$	**performed first addition**
$42 + 8 = 50$	**subtracted, and then added**

Note: At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.