# UNIT 3

## COMMON FRACTIONS

## OBJECTIVES

After completion of this unit, you should be able to solve mathematics problems involving the addition, subtraction, multiplication, and division of common fractions. This knowledge will be evidenced by correctly completing the Assignment Sheets and by scoring a minimum of 70% on the Unit Test.

Specifically, you should be able to:

1. Describe the fundamentals of common fractions.
2. Add common fractions.
3. Subtract common fractions.
4. Multiply common fractions.
5. Divide common fractions.
6. Perform combined operations with common fractions.

Each of these objectives is covered in the pages that follow.

# OBJECTIVE 1

## FUNDAMENTALS OF COMMON FRACTIONS

**Note:** It is expected that you will complete the Assignment Sheets and test for this unit without the aid of a calculator.

A **fraction** is a number that expresses part of a whole. Suppose that you order a pizza (*Figure 3.1*) at your training center and, by the time you get to the box, only one of the eight pieces remains, then you have only part of the whole pizza. Fractions are used every day by BAC Craftworkers. They appear on drawings, tape measures, rulers, etc.

### WRITING AND READING FRACTIONS

Fractions are written in the form of **a/b** or **a ÷ b** (where both **a** and **b** are whole numbers, and the number **b** is not 0). When you divide something into 2 equal parts, and pick up 1 of the 2 parts, then you have 1 out of 2 or ½. The fraction is ½, or one–half.

Figure 3.1 A Piece or "Fraction" of a Pizza

**Common fractions** are written with one number over another number. The top number is the **numerator**, while the bottom number is the **denominator**. In the following example, the fraction is read as "three–fourths," meaning that we have three out of the four parts of the whole.

$$\frac{3}{4} \quad \begin{matrix}\textbf{numerator}\\ \textbf{denominator}\end{matrix}$$

This fraction may also be written ¾. In this unit, we will write fractions using this format.

**Note:** As most fractions you will work with involve measurements, the majority of the fractions in this unit will be those used on the job (e.g., fractional parts of an inch).

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## PROPER FRACTIONS

A fraction is another form of division. A **proper fraction** is a form of division in which the result has a value smaller than 1.

**$\frac{1}{2}$ = 1 ÷ 2 = one half**

**$\frac{3}{8}$ = 3 ÷ 8 = three-eighths**

**$\frac{21}{32}$ = 21 ÷ 32 = twenty-one thirty-seconds**

## IMPROPER FRACTIONS AND MIXED NUMBERS

An **improper fraction** is a fraction in which the numerator is equal to or greater than the denominator. When an improper fraction is expressed as a combination of a whole number and a fraction, it is known as a **mixed number**. Review the following examples to see how improper fractions are converted to mixed numbers.

**$\frac{11}{8}$ = 11 ÷ 8 = 1 with a remainder of 3 = $1\frac{3}{8}$ = one and three-eighths**

**$\frac{13}{4}$ = 13 ÷ 4 = 3 $\frac{1}{4}$**

**$\frac{19}{16}$ = 19 ÷ 16 = 1 $\frac{3}{16}$**

## EQUIVALENT FRACTIONS

If the numerator and denominator of a fraction are multiplied by the same amount (or number), the value of the fraction remains unchanged. Note that $\frac{2}{2}$ = 2 ÷ 2 = 1, so multiplying a fraction by $\frac{2}{2}$ (or any other fraction where the numerator and denominator are the same) will not change the value of the fraction.

As we will see later in this unit, to multiply two fractions, you must multiply the two numerators and then the two denominators. This is shown in the following examples.

**$\frac{3}{4} \times \frac{2}{2}$ = 3 × $\frac{2}{4}$ × 2 = $\frac{6}{8}$. So, $\frac{3}{4} = \frac{6}{8}$.**

Showing this problem in columns may make it easier to understand:

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

**$\frac{7}{8} \times \frac{4}{4} = \frac{28}{32}$. So, $\frac{7}{8} = \frac{28}{32}$.**

**$\frac{3}{16} \times \frac{4}{4} = \frac{12}{64}$. So, $\frac{3}{16} = \frac{12}{64}$.**

## CONVERTING WHOLE NUMBERS TO FRACTIONS

There will be times when it will be helpful to convert a whole number to a fraction. For instance, assume you have the number 6 and would like to convert this into eighths. In other words, you want to determine how many eighths are contained in the number 6. As seen in this example, to make this conversion, simply multiply 6 by $\frac{8}{8}$.

$6 \times \frac{8}{8} = \frac{48}{8}$

So, we have 48 eighths in the number 6.

Showing this problem in columns may make it easier to understand:

$$\frac{6}{1} \times \frac{8}{8} = \frac{48}{8}$$

Here are some additional examples:

Convert 5 to sixteenths: $5 \times \frac{16}{16} = \frac{80}{16}$

Convert 9 to thirty-seconds: $9 \times \frac{32}{32} = \frac{288}{32}$

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## CONVERTING IMPROPER FRACTIONS

In the previous examples, we converted whole numbers to improper fractions. To solve some problems, you will need to do the reverse, which is to convert improper fractions to whole or mixed numbers. Review the following examples.

**Examples:** Convert $\frac{12}{4}$ to a whole or mixed number. As was mentioned previously, a fraction indicates division. So, $\frac{12}{4} = 12 \div 4 = 3$.

Convert $\frac{15}{8}$ to a whole or mixed number. $\frac{15}{8} = 15 \div 8 = 1$ with a remainder of 7, giving us the mixed number of $1\ \frac{7}{8}$.

Show that $\frac{37}{16} = 2\frac{5}{16}$.

## CONVERTING MIXED NUMBERS TO FRACTIONS

In some problems, you will need to convert a mixed number to a fraction. Review the following examples.

**Examples:** Convert 5 $\frac{3}{8}$ to eighths.

As seen previously, to convert 5 to eighths, we multiply 5 × $\frac{8}{8}$. As each of the 5 units contains 8 eighths, we would have 5 × 8, or 40 eighths. This gives us $\frac{40}{8}$. In addition to these 40, we have the 3 eighths in the mixed number. As 40 + 3 = 43, we have $\frac{43}{8}$. This shows that $5\frac{3}{8} = \frac{43}{8}$.

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Convert 7 $\frac{3}{4}$ to fourths.

$7 \times \frac{4}{4} = \frac{28}{4} + \frac{3}{4} = \frac{31}{4}$. So, $7\ \frac{3}{4} = \frac{31}{4}$.

Show that $3\ \frac{11}{16} = \frac{59}{16}$.

Show that $7\ \frac{23}{32} = \frac{247}{32}$.

## REDUCING FRACTIONS

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Some fractions can be simplified or reduced to their lowest form. For example, $\frac{6}{8}$ can be reduced to $\frac{3}{4}$. To find out if a fraction can be simplified, determine if the numerator and denominator can be divided by the same whole number. **The number selected to reduce the fraction by must produce a whole number for both the numerator and the denominator.**

In this example, both 6 and 8 can be divided by 2. If both the numerator and denominator are divided by the same number, then the value of the fraction does not change.

$$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

## CHECKING DIVISIBILITY BY 2

In some problems, the number you select to divide into the numerator and denominator will be obvious. When it is not, first try the number 2. Divide them both by 2. Then do it again and again until 2 will no longer divide into both the numerator and denominator. If 2 will not work, then try 3, 5, 7, and 11 (the numbers in between these numbers are all **divisible** by 2 or 3).

**Examples:** Reduce $^{12}/_{16}$ to its lowest form. Since 12 and 16 can both be divided by 4, $^{12}/_{16} = ^{3}/_{4}$.

Reduce $^{32}/_{64}$. Start by dividing both 32 and 64 by 2, giving us $^{16}/_{32}$. Dividing by 2 again gives us $^{8}/_{16}$. Divide by 2 once more, and we have $^{4}/_{8}$. Two more times, and we have $^{1}/_{2}$. Of course, if we had recognized that 32 would divide into both numbers, we could have gone directly to $^{1}/_{2}$.

Reduce $^{21}/_{49}$. As these are odd numbers, we are unable to divide by 2. If we try 3, we will find that will not work, neither will 5. What about 7? Bingo! If we divide by 7, we find that we have $^{3}/_{7}$.

Show that $^{18}/_{32} = ^{9}/_{16}$.

Show that $^{34}/_{64} = ^{17}/_{32}$.

**Note:** The numbers described above that are used to reduce the fractions to their lowest terms are known as **prime numbers**. A prime number is a number that is only divisible by itself and 1. Examples of prime numbers include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc.

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## INCREASING THE DENOMINATOR

Solving some problems may require that you increase the denominator (the opposite of finding the fraction's lowest form). For example, you would like to convert $\frac{3}{4}$ to thirty-seconds (32nds). By dividing 32 by 4, you see that you would need to multiply both the numerator and denominator by 8 in order to make this conversion.

$$\frac{3}{4} \times \frac{8}{8} = \frac{24}{32}$$

So, $\frac{3}{4} = \frac{24}{32}$. Review these additional examples.

**Examples:** Convert $\frac{5}{16}$ to 64ths. Since you would need to multiply 16 × 4 to get 64, you then multiply both the numerator and denominator by 4. Thus, $\frac{5}{16} \times \frac{4}{4} = \frac{20}{64}$.

Show that $\frac{15}{16} = \frac{30}{32}$.

Show that $\frac{3}{16} = \frac{12}{64}$.

## FRACTIONS AND MEASUREMENT

One of the most common uses of fractions for a BAC Craftworker is in measurement. Measurement of length (also known as **linear measure**) is done using two basic systems. The one used most often in the United States uses the inch as the basic unit of measurement. The system used in Canada and many other countries is the metric system (the metric unit of the *Advanced Mathematics* course provides more information on metrics).

## MEASURING TOOLS

The basic tool used to measure length is the ruler or tape measure (*Figure 3.2*). It is divided into feet and inches and/or metric measurements. The type of tape measure shown in *Figure 3.2* may not be used that often by Craftworkers; however, it does

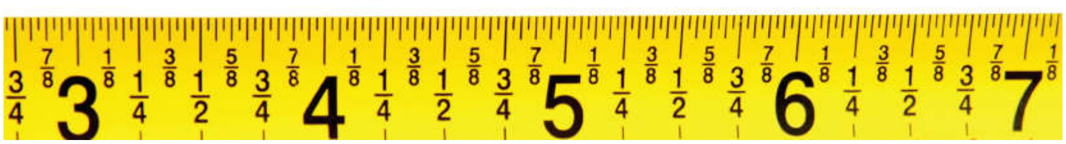


**Figure 3.2** Tape Measure with both American and Metric Measures

show the fractional parts of an inch.

Each inch is divided into parts (or fractions). These include:

- Halves ($\frac{1}{2}$)
- Quarters ($\frac{1}{4}$)
- Eighths ($\frac{1}{8}$)
- Sixteenths ($\frac{1}{16}$)
- Thirty Seconds ($\frac{1}{32}$ – only shown on some rulers and tapes)
- Sixty Fourths ($\frac{1}{64}$ – only shown on some rulers and tapes)

*Figure 3.3* shows a tape measure with the top scale divided into sixteenths and the bottom scale divided into thirty-seconds. Note that at the inch mark there is a long line that runs across the tape. The next longest line is the half–inch mark, then the eighth marks. The shortest on the top of the tape are the sixteenth marks. The shortest along the bottom of the tape are the thirty-second marks.

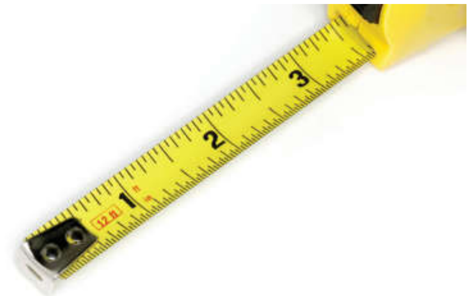

**Figure 3.3** Tape Measure with Sixteenths and Thirty Seconds

*Figure 3.4* shows a tape measure with American measurements on the top of the tape and metric measurements along the bottom of the tape.

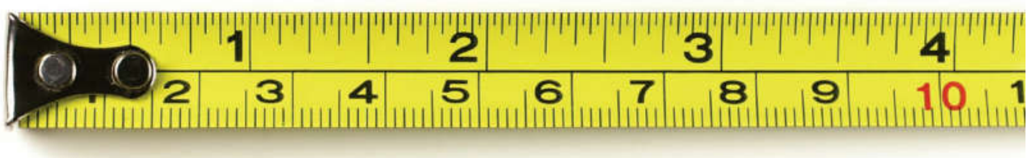


**Figure 3.4** Tape Measure with both American and Metric Measures

*Figure 3.5* shows the fractional parts of an inch down to sixteenths.

Can you locate the following fraction and mixed number measurements on the portion of ruler shown in *Figure 3.6*? Note that you indicate inches by writing the word *inches* or by using the symbol for an inch (").

Locate $\frac{3}{16}$", $\frac{7}{8}$", $1\frac{3}{8}$", $1\frac{1}{2}$", $1\frac{15}{16}$", $2\frac{1}{4}$", $3\frac{1}{16}$".

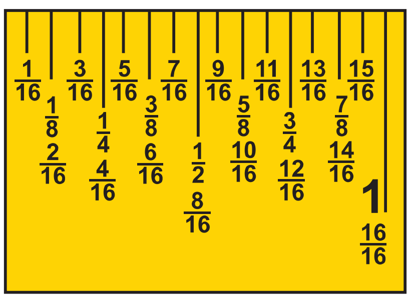


**Figure 3.5** Parts of an Inch

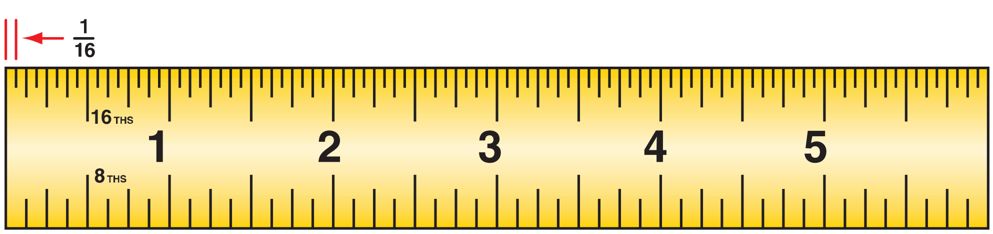


**Figure 3.6** Portion of a Ruler

> **Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 2

## ADDITION OF COMMON FRACTIONS

When measuring on the jobsite, BAC Craftworkers often need to add common fractions. For example, suppose you measure two walls for which you will be installing a plaster cornice. One wall is 96 ½" in length and the other is 112 ¾" in length. What is the total length of the two walls?

> **Note:** The rule when adding common fractions is that you must have same or "common" denominators.

### ADDING FRACTIONS WITH COMMON DENOMINATORS

To add fractions that have the same denominator, simply add the numerators. If you have 3 apples and 2 more, then you have 5 apples. The same applies to fractions. If you mark a line with a ruler that is $3/16$" in length, and then add 5 more 16ths, then you would have $3/16" + 5/16" = 8/16"$. This sum of $8/16"$ can be reduced to ½".

**$5/32 + 23/32 = 28/32 = 7/8$ reduced to lowest terms**

**$21/64 + 13/64 = 34/64 = 17/32$ reduced to lowest terms**

### ADDING FRACTIONS WITH DIFFERENT DENOMINATORS

When you want to add fractions that have different denominators, you must change all of the denominators so that they are the same. This is known as finding the **lowest common denominator** (LCD).

### DETERMINING THE LCD

To determine the LCD, find the smallest (or lowest) number into which all denominators can be divided. Note that if all of the fractions in a problem are found on the ruler or tape measure (e.g., ½, ¼, ⅛, $1/16$, $1/32$, $1/64$), the LCD will be the smallest fraction or the denominator with the largest number. Review these examples.

> **Note:** Think about this: The smallest fraction is the denominator with the largest number. Since the denominator is being divided into the numerator, the larger the denominator, the smaller the fraction (e.g., 1/64 is a smaller amount than 1/4).

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**Examples:** Add $\frac{1}{2}$" + $\frac{1}{4}$" + $\frac{1}{8}$".

Sometimes the LCD will be one of the denominators. In this example, it is clear that 2 and 4 can be divided into 8. So, you now convert each of the fractions to eighths:

$\frac{1}{2}$" = $\frac{4}{8}$"

$\frac{1}{4}$" = $\frac{2}{8}$"

**Answer:** $\frac{4}{8}$" + $\frac{2}{8}$" + $\frac{1}{8}$" = $\frac{7}{8}$"

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Add $\frac{3}{4}$" + $\frac{3}{8}$" + $\frac{11}{16}$".

By inspection, we determine that the LCD is 16.

**Answer:** $\frac{12}{16}$" + $\frac{6}{16}$" + $\frac{11}{16}$" = $\frac{29}{16}$" = 1 $\frac{13}{16}$"

**Note:** Determining the LCD when it is not obvious by inspection is a bit more complicated. In this unit, we will only use the inspection method for determining the LCD.



## ADDING MIXED NUMBER FRACTIONS

When adding mixed numbers, you must first add the fractions. Often, this will mean determining the LCD and then adding the fractions. The sum of the fractions is then added to the whole numbers in the problem. Carefully review the following examples.

**Example:** Add 6 $\frac{3}{4}$" + 7 $\frac{5}{16}$".

First add the fractions ($\frac{3}{4}$ + $\frac{5}{16}$). By inspection, we determine that the LCD is 16. We now convert $\frac{3}{4}$ to 16ths. To convert 4 to 16, we multiply by 4. We must do the same for the numerator. This looks like:

$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$

Add the fractions: $\frac{12}{16} + \frac{5}{16} = \frac{17}{16} = 1\,\frac{1}{16}$

Add the sum of the fractions to the whole numbers.

**Answer:** 6 + 7 +1 $\frac{1}{16}$ = 14 $\frac{1}{16}$"

**Example:** Add $5\ \frac{13}{16} + 18\ \frac{41}{64} + 17\ \frac{19}{32}$.

By inspection, the LCD = 64.

It may be helpful to see this problem in columns:

$$\begin{array}{rrcr} & 5\ \frac{13}{16} & = & 5\ \frac{52}{64} \\ + & 18\ \frac{41}{64} & = & 18\ \frac{41}{64} \\ + & 17\ \frac{19}{32} & = & 17\ \frac{38}{64} \\ \hline \end{array}$$

Converting the fractions to 64ths produces $\frac{13}{16} = \frac{52}{64}$ and $\frac{19}{32} = \frac{38}{64}$

Add the fractions: $\frac{52}{64} + \frac{41}{64} + \frac{38}{64} = \frac{131}{64} = 2\ \frac{3}{64}$

Add the sum of the fractions to the whole numbers.

**Answer:** $5 + 18 + 17 + 2\ \frac{3}{64} = 42\ \frac{3}{64}$

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 3

## SUBTRACTION OF COMMON FRACTIONS

To subtract any two fractions, you must have common denominators. To subtract fractions that have the same denominator, simply subtract the numerators and simplify the answer as needed.

$17/32 - 11/32 = 6/32 = 3/16$

It may be helpful to see this problem in columns:

$$\begin{array}{r} 17/32 \\ -\ 11/32 \\ \hline 6/32 \end{array} = 3/16$$

$5/8 - 3/8 = 2/8 = 1/4$

$53/64 - 15/64 = 38/64 = 19/32$

To subtract mixed numbers where the denominators of the fractions are the same, first subtract the fractions, then the whole numbers.

**Example:** Subtract 4 ¾ – 1¼.

**Step 1:** Subtract the fractions and reduce the result.

¾ – ¼ = 2⁄4 = ½

**Step 2:** Subtract the whole numbers.

4 – 1 = 3

**Step 3:** Combine the results.

4 ¾ – 1 ¼ = 3 ½

**Answer:** 3 ½

**Example:** Subtract 17 ⅞ – 11 ⅛.

**Step 1:** Subtract the fractions and reduce the result.

⅞ – ⅛ = 6⁄8 = ¾

**Step 2:** Subtract the whole numbers.

17 – 11 = 6

**Step 3:** Combine the results.

17 ⅞ – 11 ⅛ = 6 ¾

**Answer:** 6 ¾

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## SUBTRACTING WITH BORROWING

In Unit 2, we learned that if the number being subtracted is larger than the number it is being subtracted from, then we need to "borrow" from the next column. That same rule applies to subtracting mixed numbers. Review these examples carefully.

**Example:** Subtract $15\ \frac{5}{8} - 3\ \frac{7}{8}$.

Since $\frac{7}{8}$ is larger than $\frac{5}{8}$, we need to borrow 1 from 15. Convert the 1 to eighths (1 = 8 eighths, or 8/8), and add this to the $\frac{5}{8}$. This gives us $\frac{13}{8}$. The problem now becomes:

$$\begin{array}{r} 15\ \frac{5}{8} \\ -\ 3\ \frac{7}{8} \\ \hline \end{array} = \begin{array}{r} 14\ \frac{13}{8} \\ -\ 3\ \frac{7}{8} \\ \hline \end{array}$$

After borrowing and reducing, the problem now looks like this:

$14\ \frac{13}{8} - 3\ \frac{7}{8} = 11\ \frac{6}{8} = 11\ \frac{3}{4}$

**Answer:** $11\ \frac{3}{4}$

**Example:** Subtract $34\ \frac{15}{32} - 28\ \frac{23}{32}$.

After borrowing $\frac{32}{32}$ from 34, the problem becomes:

$$\begin{array}{r} 34\ \frac{15}{32} \\ -\ 28\ \frac{23}{32} \\ \hline \end{array} = \begin{array}{r} 33\ \frac{47}{32} \\ -\ 28\ \frac{23}{32} \\ \hline \end{array}$$

$33\ \frac{47}{32} - 28\ \frac{23}{32} = 5\ \frac{24}{32} = 5\ \frac{3}{4}$

**Answer:** $5\ \frac{3}{4}$

## SUBTRACTING WITH DIFFERENT DENOMINATORS

To subtract fractions that have different denominators, first convert the fractions to the same denominators using the LCD. Then, perform the subtraction as in the previous examples.

**Examples:** Subtract $\frac{9}{16} - \frac{3}{8}$.

By inspection, we determine that the LCD is 16. We then convert the $\frac{3}{8}$ to 16ths, giving us:

$\frac{9}{16} - \frac{6}{16} = \frac{3}{16}$

**Answer:** $\frac{3}{16}$

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Subtract $2\ \frac{1}{4} - 1\ \frac{21}{32}$.

The LCD is 32, resulting in $2\ \frac{8}{32} - 1\ \frac{21}{32}$.

After borrowing, we have $1\ \frac{40}{32} - 1\ \frac{21}{32} = \frac{19}{32}$.

**Answer:** $\frac{19}{32}$

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**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 4

## MULTIPLICATION OF COMMON FRACTIONS

The rules for multiplying fractions include:

1. Change any mixed numbers to improper fractions.
2. Multiply the numerators.
3. Multiply the denominators.
4. Reduce the product (answer) to lowest terms (if applicable). For some products, there is no common factor that will allow the fraction to be expressed in lower terms.

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**Examples:** Multiply ¾ × 9⁄16.

$$\frac{3}{4} \times \frac{9}{16} = \frac{27}{64}$$

Note that the fraction cannot be expressed in any lower terms.

⅞ × ½ = 7⁄16

⅜ × 9⁄16 = 27⁄128

¾ × ⅔ = 6⁄12 = ½

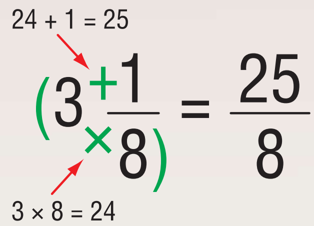
### SHORTCUT TO IMPROPER FRACTIONS

As mentioned previously, when multiplying mixed numbers, first change the mixed number to an improper fraction. Now that we have learned to multiply fractions, it is time to show a shortcut for converting mixed numbers to improper fractions.

Assume you want to multiply 3 ⅛ × 4 ¾. Both of these mixed numbers must first be converted to improper fractions. Earlier we learned that there would be 8 eighths in 1. So, we would have 24 eighths in 3. This can be done quickly by simply multiplying the denominator by the whole number and then adding the numerator. Review the following example.

**Examples:** Multiply $3\frac{1}{8} \times 4\frac{3}{4}$.

**Step 1:** Convert $3\frac{1}{8}$ to an improper fraction. To do this, you multiply $8 \times 3$, and then add 1. This is $\frac{25}{8}$. This may be easier to see using the following illustration:




**Step 2:** Convert $4\frac{3}{4}$ to an improper fraction. The sequence is $4 \times 4 + 3 = 19$. So, the answer is $\frac{19}{4}$.

So, $3\frac{1}{8} \times 4\frac{3}{4} = \frac{25}{8} \times \frac{19}{4} = \frac{475}{32} = 14\frac{27}{32}$.

Let's write this fraction in a column format to make the process more clear.

$$\frac{25}{8} \times \frac{19}{4} = \frac{475}{32} = 14\frac{27}{32}$$

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Multiply $7\frac{11}{16} \times 2\frac{1}{4}$.

**Step 1:** Convert $7\frac{11}{16}$ and $2\frac{1}{4}$ to improper fractions.

$7\frac{11}{16} = ([16 \times 7] + 11)\ /16 = \frac{123}{16}$

$2\frac{1}{4} = ([4 \times 2] + 1)/4 = \frac{9}{4}$

**Step 2:** Multiply the fractions.

$\frac{123}{16} \times \frac{9}{4} = \frac{1107}{64}$

**Step 3:** Reduce the fraction to lowest form.

$\frac{1107}{64} = 17\frac{19}{64}$

**Answer:** $17\frac{19}{64}$

Multiply 15 $\frac{7}{8}$ × 12 $\frac{19}{32}$.

**Step 1:** Convert 15 $\frac{7}{8}$ and 12 $\frac{19}{32}$ to improper fractions.

$15\frac{7}{8} = ([8 \times 15] + 7)/8 = \frac{127}{8}$

$12\frac{19}{32} = ([32 \times 12] + 19)/32 = \frac{403}{32}$

**Step 2:** Multiply the fractions.

$$\frac{127}{8} \times \frac{403}{32} = \frac{51{,}181}{256}$$

$\frac{51{,}181}{256}$ = 199 with a remainder of 237

**Step 3:** Reduce the fraction to lowest form.

199 with a remainder of 237 = 199 $\frac{237}{256}$

**Answer:** 199 $\frac{237}{256}$

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Show that 9 $\frac{11}{32}$ × 3 ½ = 32 $\frac{45}{64}$.

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### Fun with Numbers – Finding Half of a Fraction

Here is a quick way to find half of a fraction.

First, double the denominator, and use the same numerator so that half of $\frac{3}{8}$ = $\frac{3}{16}$; half of ¾ = $\frac{3}{8}$, and so on.

To find half of a mixed fraction when the whole number is an even number (e.g., 4$\frac{9}{16}$), first take half of the whole number. Then, use the same method as above ($\frac{3}{16}$ = $\frac{3}{32}$). So, half of 4$\frac{9}{16}$ = 2$\frac{9}{32}$.

When the whole number is an odd number (e.g., 5$\frac{5}{8}$), first take half of the whole number, and discard the remainder. Half of 5 is 2½. Discard the ½. Second, double the denominator again so that 8 becomes 16. Third, add the original numerator and denominator together to get the new denominator 5 + 8 = 13. Half of 5$\frac{5}{8}$ = 2$\frac{13}{16}$.

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 5

## DIVISION OF COMMON FRACTIONS

To divide fractions, you simply invert (flip) the divisor, and then multiply. Suppose that you want to divide ⅝ by ¼ (⅝ ÷ ¼).

In a column format, the problem looks like this:

$$\frac{\frac{5}{8}}{\frac{1}{4}}$$

In this problem, ¼ is the divisor (the number being divided into the dividend). When we invert, or turn, ¼ upside down, we get 4⁄1. We now multiply this by the dividend.

$$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{1} = \frac{20}{8} = 2\frac{1}{2}$$

As ¼ is smaller than ⅝, finding that ¼ will divide into ⅝ 2 ½ times seems reasonable.

**Example:** 15⁄16 ÷ ⅞

15⁄16 × 8⁄7 = 120⁄112

120⁄112 = 60⁄56 = 30⁄28 = 1 1⁄14

### DIVIDING WHOLE NUMBERS BY FRACTIONS

To divide a whole number by a fraction, write the whole number over 1 as a fraction, invert the divisor, and multiply.

**Examples:** 2 ÷ ⅞

**Step 1:** Write 2 as 2⁄1. We now have 2⁄1 ÷ ⅞.

**Step 2:** Invert and multiply.

2⁄1 ÷ ⅞ becomes 2⁄1 × 8⁄7 = 16⁄7

**Step 3:** Reduce the fraction.

16⁄7 = 2 2⁄7

**Answer:** 2 2⁄7

UNIT 3

$^{13}/_{16} \div 4$

Write 4 as $^4/_1$. We now have $^{13}/_{16} \div {}^4/_1$.

Invert and multiply.

**Answer:** $^{13}/_{16} \div {}^4/_1$ becomes $^{13}/_{16} \times {}^1/_4 = {}^{13}/_{64}$

## DIVIDING MIXED NUMBERS BY FRACTIONS

To divide a mixed number and a fraction or to divide two mixed numbers, convert the mixed numbers to improper fractions, invert the divisor, and then multiply.

**Examples:** $3\ ^9/_{16} \div {}^1/_8$

**Step 1:** Convert $3\ ^9/_{16}$ to an improper fraction.

$([16 \times 3] + 9)/16 = {}^{57}/_{16}$

**Step 2:** Invert and multiply.

$^{57}/_{16} \div {}^1/_8$ becomes $^{57}/_{16} \times {}^8/_1 = {}^{456}/_{16}$

$= 28\ ^8/_{16}$

**Answer:** $28\ ^1/_2$

---

$4\ ^{15}/_{32} \div 1\ ^1/_2$

Convert $4\ ^{15}/_{32}$ and $1\ ^1/_2$ to improper fractions.

$4\ ^{15}/_{32} = ([32 \times 4] + 15)/32 = {}^{143}/_{32}$

$1\ ^1/_2 = ([2 \times 1] + 1)/2 = {}^3/_2$

Invert and multiply.

$^{143}/_{32} \div {}^3/_2$ becomes $^{143}/_{32} \times {}^2/_3 = {}^{286}/_{96}$

$= 2\ ^{94}/_{96}$

**Answer:** $2\ ^{47}/_{48}$

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 6

## COMBINED OPERATIONS WITH COMMON FRACTIONS

When more than one of the four basic functions appears in a problem (addition, subtraction, multiplication, and division), a specific sequence must be followed in order to correctly solve the problem. This is referred to as "mathematical order of operations."

As a reminder, here are the rules for order of operations:

1. Simplify any expression inside a grouping symbol (e.g., parentheses and brackets).
2. Simplify expressions with exponents (exponents will be covered later in this manual).
3. Carry out multiplication or division from *left to right*.
4. Simplify addition or subtraction from *left to right*.

Review the following examples to see how to perform combined operations with common fractions.

**Example:** 6 ⅛ ÷ (1 ½ + ¼) – 1 ½ + 2 × 2 ½

**Step 1:** Simplify the expression inside the parentheses by adding.

6 ⅛ ÷ (1 ½ + ¼) – 1 ½ + 2 × 2 ½

1 ½ + ¼ = 1 ¾

**Step 2:** Perform the multiplication.

6 ⅛ ÷ 1 ¾ – 1 ½ + 2 × 2 ½

2 × 2 ½ = 5

**Step 3:** Perform the division.

6 ⅛ ÷ 1 ¾ – 1 ½ + 5

6 ⅛ ÷ 1 ¾ = 3 ½

**Step 4:** Perform the subtraction.

3 ½ – 1 ½ + 5

3 ½ – 1 ½ = 2

**Step 5:** Perform the addition.

2 + 5 = 7

**Answer:** 7

UNIT 3

**Example:** $\frac{3}{4} + \frac{5}{8} \times \frac{1}{2}$

**Step 1:** Multiply the two fractions.

$\frac{3}{4} + \frac{5}{8} \times \frac{1}{2}$

$\frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

**Step 2:** Convert the fractions to the LCD and add.

$\frac{3}{4} + \frac{5}{16}$

$\frac{12}{16} + \frac{5}{16} = \frac{17}{16}$

**Step 3:** Convert the answer into a mixed fraction.

$\frac{17}{16} = 1\ \frac{1}{16}$

**Answer:** $1\frac{1}{16}$.

**Example:** $(2\ \frac{11}{16} - 1\ \frac{3}{8}) \div \frac{1}{4}$

**Step 1:** Convert the expression in the parentheses to the LCD.

$(2\ \frac{11}{16} - 1\ \frac{3}{8}) \div \frac{1}{4}$

$(2\ \frac{11}{16} - 1\ \frac{6}{16})$

**Step 2:** Subtract.

$(2\ \frac{11}{16} - 1\ \frac{6}{16}) \div \frac{1}{4}$

$2\ \frac{11}{16} - 1\ \frac{6}{16} = 1\ \frac{5}{16}$

**Step 3:** Convert the mixed number to a fraction.

$1\ \frac{5}{16} \div \frac{1}{4}$

$1\ \frac{5}{16} = \frac{21}{16}$

**Step 4:** Invert and multiply.

$\frac{21}{16} \div \frac{1}{4}$

$\frac{21}{16} \times \frac{4}{1} = \frac{84}{16}$

**Step 5:** Convert the answer to a mixed number and simplify.

$\frac{84}{16} = 5\ \frac{4}{16} = 5\ \frac{1}{4}$

**Answer:** $5\ \frac{1}{4}$.

UNIT 3

**Example:** $3\ ^{3}/_{16} \div {}^{1}/_{4} + 4\ ^{5}/_{8}$

**Step 1:** Convert to improper fraction.

$3\ ^{3}/_{16} \div {}^{1}/_{4} + 4\ ^{5}/_{8}$

$3\ ^{3}/_{16} = {}^{51}/_{16}$

**Step 2:** Invert, multiply, and reduce to improper fraction.

${}^{51}/_{16} \div {}^{1}/_{4} + 4\ ^{5}/_{8}$

${}^{51}/_{16} \times {}^{4}/_{1} = {}^{204}/_{16}$

**Step 3:** Repeatedly divide by 2 to reduce.

${}^{204}/_{16} + 4^{5}/_{8} =$

${}^{51}/_{4} + 4\ ^{5}/_{8}$

**Step 4:** Find common denominators.

${}^{51}/_{4} + 4\ ^{5}/_{8} =$

$12\ ^{6}/_{8} + 4\ ^{5}/_{8}$

**Step 5:** Add the mixed numbers.

$12\ ^{6}/_{8} + 4\ ^{5}/_{8} = 16\ ^{11}/_{8} = 17\ ^{3}/_{8}$

**Answer:** $17\ ^{3}/_{8}$

**Example:** $5\ ^{2}/_{3}\ (3\ ^{5}/_{12} + {}^{3}/_{4} \times {}^{1}/_{2})$

**Step 1:** Multiply fractions.

$5\ ^{2}/_{3}\ (3\ ^{5}/_{12} + {}^{3}/_{4} \times {}^{1}/_{2})$

${}^{3}/_{4} \times {}^{1}/_{2} = {}^{3}/_{8}$

**Step 2:** Convert to LCD.

$5\ ^{2}/_{3}\ (3\ ^{5}/_{12} + {}^{3}/_{8})$

$3\ ^{5(2)}/_{12(2)} + {}^{3(3)}/_{8(3)} =$

$3\ ^{10}/_{24} + {}^{9}/_{24}$

**Step 3:** Add and reduce fraction.

$5\ ^{2}/_{3}\ (3\ ^{10}/_{24} + {}^{9}/_{24})$

$3\ ^{10}/_{24} + {}^{9}/_{24} = 3\ ^{19}/_{24}$

UNIT 3

**Step 4:** Convert to improper fractions.

$5\ \frac{2}{3}\ (3\ \frac{19}{24}) =$

$\frac{17}{3} \times \frac{91}{24} = \frac{1547}{72} = 21\ \frac{35}{72}$

**Answer:** $21\ \frac{35}{72}$

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

UNIT 3

# UNIT 4

## DECIMAL FRACTIONS

## OBJECTIVES

After completion of this unit, you should be able to solve mathematics problems involving the addition, subtraction, multiplication, and division of decimal fractions. This knowledge will be evidenced by correctly completing the Assignment Sheets and by scoring a minimum of 70% on the Unit Test.

Specifically, you should be able to:

1. Use a calculator to add, subtract, multiply, and divide whole numbers and decimal fractions.
2. Describe the fundamentals of decimal fractions.
3. Add decimal fractions.
4. Subtract decimal fractions.
5. Multiply decimal fractions.
6. Divide decimal fractions.
7. Perform combined operations with decimal fractions.

Each of these objectives is covered in the pages that follow.

UNIT 4

# OBJECTIVE 1

## USE A CALCULATOR

Beginning in this unit, you will be able to use a calculator to solve Assignment Sheet and test problems. In order to complete the problems in this and other units, you will need a scientific calculator. Scientific calculators are readily available and are fairly inexpensive. Your instructor may be able to recommend a specific calculator, as this unit will be easier if all of the students are using the same type and model of scientific calculator.

**Figure 4.1** Scientific Calculator

### THE SCIENTIFIC CALCULATOR

A scientific calculator (*Figure 4.1*) is a type of electronic calculator that will allow you to work with exponents, roots, and other functions (e.g., trigonometric functions as covered in *Advanced Mathematics*). Although you can use a very inexpensive calculator to add, subtract, multiply, and divide whole numbers and decimal fractions, this kind of calculator will typically not have the more advanced functions found on a scientific calculator.

> **Note:** Not all scientific calculators operate alike. Some newer models process data in a way that is similar to how we speak. For example, the sine of 45 degrees is entered by pressing the sine function and then 45. Older styles process the same problem by requiring the user to enter 45, and then press the sine function. To learn the specific sequences for your calculator, always refer to the manufacturer's instruction manual.

UNIT 4

## THE CONSTRUCTION CALCULATOR


Figure 4.2 Construction Calculator

Many BAC Craftworkers carry a construction calculator (*Figure 4.2*). This calculator typically allows you to easily determine precise angle measurements and solve complex design and construction-math problems (including trigonometric functions, 3-4-5 right triangles, and estimating materials and costs). You will learn to use a construction calculator in this manual.

Solving a problem using your calculator is a matter of entering numbers and pressing the correct calculator keys in the right sequence. If you enter incorrect information or press an incorrect key, your calculator may still provide you with an answer – but it will be the wrong answer.

## CALCULATOR KEYSTROKE BASICS

Learning to use your calculator requires practice. *Figure 4.3* shows the sequence to solve the problem 2 × 5.

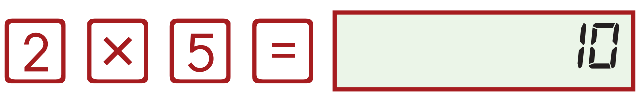


Figure 4.3 Sample Calculator Keystrokes

The sequence of keystrokes is:

1. Enter 2.
2. Press the × key.
3. Enter 5.
4. Press the = key.

After performing this sequence, the answer 10 appears on the calculator screen.

UNIT 4

## CALCULATOR KEYSTROKE PRACTICE

Let's look at some problems similar to the ones completed in previous units – only this time we will use the calculator Remember that each box represents a different key for you to press.

**Examples:** What is the combined weight of two panels? One weighs 150 pounds, and the other weighs 256 pounds.

[1] [5] [0] [+] [2] [5] [6] [=] 406

---

You have 27 fasteners, and someone picks up 11 of them. How many are remaining?

[2] [7] [–] [1] [1] [=] 16

---

Bill works 46 hours a week for 8 weeks. How many hours has he worked?

[4] [6] [×] [8] [=] 368

---

Simplify 12 + 3 (8 – 5) + 16.

[1] [2] [+] [3] [×] [(] [8] [–] [5] [)]
[+] [1] [6] [=] 37

> **Note:** Scientific calculators use each key twice. This means that each key can serve two different functions. There will be a key on your calculator that allows you to "switch" to the second level of functions. Refer to the manufacturer's instruction guide for details. In this manual, we will not show how to get to the second functions, as the procedure will differ with each calculator.

It is recommended that you go back to the examples and (if available) the Assignment Sheets in the previous units, and use your calculator to solve some of the problems. This will help you to learn to use your calculator to perform addition, subtraction, multiplication, and division with whole numbers and common fractions.

> **Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 2

## FUNDAMENTALS OF DECIMAL FRACTIONS

The number 1 is a whole number. It is a complete number and can be thought of as representing a complete unit of something (e.g., length, weight, number of bolts). Because 1 is a whole number, every number that is a multiple of 1 is also a whole number (e.g., 2, 3, 4, 5, 6, etc.).

A fraction is part of a whole. The common fraction ¾ indicates that there are 4 parts of something and that we have 3 of the parts.

A fraction is an instruction to perform division. To understand this better, review the following steps.

**Step 1:** The fraction ¾ is an instruction to divide 3 by 4 or 3 ÷ 4. As seen below, 4 cannot be divided into 3. So, we place a **decimal point** after the 3. We can place one or more zeros (0) following the decimal point without changing the value of the number.

4 ) 3.00 ← **decimal point**

**Step 2:** Because 4 cannot be divided into 3, we must divide 4 into 30. Note in the example that we also place a decimal point in the quotient, or answer.

```
     .7
4 ) 3.00
```

**Step 3:** Multiply 7 in the quotient times the divisor (4). This is 4 × 7 = 28. Record the 28 under the 30.

```
     .7
4 ) 3.00
   – 28
```

**Step 4:** Subtract 30 – 28 = 2. Bring down the "0" to make this 20.

```
     .7
4 ) 3.00
   – 28
      20
```

**Step 5:** We now want to divide 4 into 20. This gives us 5.

```
      .75
 4 ) 3.00
    – 28
       20
```

**Step 6:** Multiply 5 in the quotient times the divisor (4). This is 4 × 5 = 20. Record the 20 under the 20.

```
      .75
 4 ) 3.00
    – 28
       20
     – 20
        0
```

**Step 7:** Completing the division process, we see that ¾ = 0.75. The value 0.75 is known as a **decimal fraction**. A decimal fraction may be written as 0.75 or as .75. The 0 is placed in front of (or to the left of) the decimal point to ensure that the number is seen as a decimal fraction.

BAC Craftworkers use decimal fractions or numbers every day (e.g., odometer on trucks to indicate mileage, change for a dollar, and measurements on drawings). *Figure 4.4* shows a construction calculator displaying 4 feet and 8¾ inches. Also shown is the same calculator displaying the decimal equivalent (4.729167 feet). *Figure 4.5* shows a distance–measuring instrument indicating a distance of 35.870 meters (a metric measurement).


**Figure 4.4** Construction Calculator Displaying Feet and Inches and the Decimal Equivalent


**Figure 4.5** Distance Measure Indicating Meters

## POSITION AND DECIMAL NUMBER VALUE

As we learned in a previous unit, the value of a whole number is dependent on its columnar position, or position (tens, hundreds, thousands, etc.) within the number. The same applies to the value of a decimal fraction or number. The column or place values for whole numbers that appear to the "left of the decimal point" are shown in *Table 4.1*.

| Place Values for Whole Numbers | | | | | | | |
|---|---|---|---|---|---|---|---|
| Millions | Hundred-Thousands | Ten-Thousands | Thousands | Hundreds | Tens | Units (or Ones) | Decimal Point |
| 3 | 5 | 8 | 4 | 9 | 6 | 2 | . |

**Table 4.1** Place Values for Whole Numbers

The decimal point and decimal fractions appear to the "right" of the units, or ones, column. The place values for decimal fractions are shown in *Table 4.2*.

| Place Values for Decimal Fractions | | | | | | |
|---|---|---|---|---|---|---|
| Decimal Point | Tenths | Hundredths | Thousandths | Ten-Thousandths | Hundred-Thousandths | Millionths |
| . | 1 | 6 | 3 | 5 | 7 | 4 |

**Table 4.2** Place Values for Decimal Fractions

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The decimal fraction in *Table 4.2* is written as 0.163574. Just like whole numbers, this decimal number may also be written with spaces (beginning to the right of the decimal point and moving to the right) to group the numbers in threes. This means that our decimal number could look like 0.163 574. In this unit, we will write decimal fractions without spaces. The number in *Table 4.2* is read as "one hundred sixty-three thousand five hundred seventy-four millionths".

The decimal fraction or number having the greatest value is the one immediately following the decimal point. As you move to the right, the numbers become smaller.

**1 tenth = 10 hundredths = 100 thousandths**

Here are some examples of decimal fractions:

0.1 = 1 tenth
0.25 = 25 hundredths
0.625 = 625 thousandths
0. 8045 = 8,045 ten-thousandths

> **Note:** While most work performed by BAC Craftworkers on the job will involve decimal fractions that are no smaller than thousandths, in this unit, we will work with some problems with smaller numbers in order to help you develop needed math skills.

## WRITING MIXED NUMBERS AS DECIMAL FRACTIONS

A decimal fraction may also be written as a mixed number. For example, the mixed number 1½ may be written as 1.5 (once you convert the ½ to 0.5 by dividing 1 ÷ 2).

**3 ¾ = 3.75**

**5 ⅝ = 5.625**

**12 11⁄16 = 12.6875**

## ROUNDING WITH DECIMAL FRACTIONS

When converting fractions to decimal fractions, there will be situations when the quotient appears as if it will go on forever (and sometimes it will). Obviously, the further you move to the right, the smaller the number becomes. Depending on the precision required, you may need to round a number to a specific value.

For example, suppose you want to convert the fraction 5⁄7 to a decimal fraction. When you begin to divide 5 by 7, you find that the answer is 0.714285714, and it just keeps going. You want to round this number to a specific decimal place. Here are the rules to round numbers.

1. Locate the place (value) to be rounded, and look at the number immediately to the right of this number.
2. If the number occupying this place is 4 or less, the number to be rounded remains the same, and all numbers to the right of this number are dropped or removed.
3. If the number occupying this place is 5 or more, the number to be rounded is increased by 1, and all numbers to the right of this number are dropped or removed.

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**Examples:** Round 0.714285714 to the nearest thousandth (third decimal place). As we are rounding to 3 places, we look at the 4. The number to the right of the 4 is 2. So, we leave the 4, and drop the other numbers. This gives us 0.714.

Round 0.6875 to the nearest hundredth (second decimal place). We look at the number to the right of the 8 and see that it is a 7. We then increase the 8 to 9, and then drop the other numbers. This gives us 0.69.

Round 3.1695 to the nearest thousandth. Show that the answer is 3.170, or 3.17.

Round 6.549 to the nearest tenth. Show that the answer is 6.5.

**Examples:** Here are some additional examples.

Convert $1\frac{7}{8}$ to a decimal fraction.

The process is: 1 + (7 ÷ 8) = 1.875

Note that you can use your calculator to solve this problem:

[1] [+] [7] [÷] [8] [=] [1.875]

Convert 4 $\frac{3}{4}$ to a decimal fraction.

The process is: 4 + 3 ÷ 4 = 4.75

Convert 2 15/16 to a decimal fraction rounded to the nearest thousandth (3 decimal places).

The process is: 2 + 15 ÷ 16 = 2.9375 or 2.938

Convert 8 $\frac{3}{32}$ to a decimal fraction rounded to the nearest ten-thousandth (4 decimal places).

The process is: 8 + 3 ÷ 32 = 8.09375 = 8.0938

Convert 12 $\frac{53}{64}$ to a decimal fraction rounded to the nearest hundredth (2 decimal places).

The process is: 12 + 53 ÷ 64 = 12.828125 = 12.83

## FINDING THE COMMON FRACTION EQUIVALENT

The common fraction equivalent of a decimal fraction is found by expressing the decimal as a common fraction and then reducing it to its lowest terms.

**Examples:** Convert 0.8 to a common fraction.

The process is: $0.8 = \frac{8}{10} = \frac{4}{5}$

Convert 1.25 to a common fraction.

The process is: $1.25 = 1 + 0.25$

$1 + \frac{25}{100} = 1\frac{1}{4}$

Convert 17.4375 to a common fraction.

The process is: $17.4375 = 17 + \frac{4375}{10000}$. To reduce $\frac{4375}{10000}$ to lowest terms, repeatedly divide both the numerator and denominator by 5. The result is $\frac{7}{16}$. So, $17.4375 = 17\,\frac{7}{16}$.

Note that if you are using a construction calculator, you may be able to do these conversions directly. The key sequence to perform the previous problem may look similar to the following:

[1] [7] [•] [4] [3] [7] [5] [CONVERT] [INCH] [=]

[17 7/16]

*Figure 4.6* shows the initial entry of 17.4375 and the conversion to 17 $\frac{7}{16}$. For specific details on how to perform this conversion, refer to the manufacturer's instructions.


**Figure 4.6** Construction Calculator Performing a Conversion

> **Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 3

## ADDITION OF DECIMAL FRACTIONS

Adding decimals is essentially the same as adding whole numbers. The difference is that we now have decimal numbers. Here are the steps for solving addition problems.

1. Align the decimal points and place values in vertical columns.

2. Beginning on the far right, add the numbers in each column. Just as in addition of whole numbers, carry the values to the column to the left.

Review the following examples.

**Example:** 0.75 + 0.375 (¾ + ⅜)

**Step 1:** Align decimal points. In order to do so, you must add a "0" after the "5" in .75.

$$\begin{array}{r} .750 \\ +\ .375 \\ \hline \end{array}$$

**Step 2:** Beginning on the right, add the numbers in the thousandths column. In this example, we add 5 and 0. The sum is 5.

$$\begin{array}{r} .750 \\ +\ .375 \\ \hline 5 \end{array}$$

**Step 3:** Add the numbers in the hundredths column. We have 5 + 7 = 12. As we only have space for the 2, we carry the 1 to the top of the next column.

$$\begin{array}{r} 1\phantom{00} \\ .750 \\ +\ .375 \\ \hline 25 \end{array}$$

**Step 4:** Add the numbers in the tenths column. We have 7 + 3 + the 1 we carried for a total of 11.

$$\begin{array}{r} 1\phantom{00} \\ .750 \\ +\ .375 \\ \hline 1125 \end{array}$$

UNIT 4

**Step 5:** Bring the decimal point down.

```
   1
  .750
+ .375
------
 1.125
```

**Example:** 1.873 + 2.128

**Step 1:** Align decimal points.

```
  1.873
+ 2.128
-------
```

**Step 2:** Beginning on the right, add the numbers in the thousandths column. In this example, we add 3 and 8. The sum is 11. As we only have space for the 3, we "carry" the 1 to the top of the next column.

```
    1
  1.873
+ 2.128
-------
      1
```

**Step 3:** Add the numbers in the hundredths column. We have 7 + 2 + the 1 we carried for a total of 10. Again, since we only have space for the 0, we must carry the 1 to the top of the next column.

```
   11
  1.873
+ 2.128
-------
     01
```

**Step 4:** Add the numbers in the tenths column. We have 8 + 1 + the 1 we carried for a total of 10. With only space for the 0, let's carry the 1 to the top of the next column.

```
  1 11
  1.873
+ 2.128
-------
    001
```

**Step 5:** Add the numbers in the units, or ones, column. We have 1 + 2 + the 1 we carried for a total of 4.

```
  1 11
  1.873
+ 2.128
-------
  4 001
```

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**Step 6:** Bring down the decimal point.

$$\begin{array}{r} 1\ 11\phantom{0} \\ 1.873 \\ +\ 2.128 \\ \hline 4.001 \end{array}$$

Let's do the previous problem using your calculator.

1 · 8 7 3 + 2 · 1 2 8 = 4.001

Use your calculator to check that the following answers are correct.

**13.875 + 7.096 = 20.971**

**5¾ + 6⅞ = 5.75 + 6.875 = 12.625**

**4.892 + 0.67 + 1.203 = 6.765**

**10.7854 + 2.386 + 17.0942 = 30.2656**

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

UNIT 4

# OBJECTIVE 4

## SUBTRACTION OF DECIMAL FRACTIONS

Subtracting decimals is essentially the same as subtracting whole numbers. Here are the steps for solving subtraction problems.

1. Align the decimal points and place values in vertical columns.

2. Beginning on the far right, subtract the numbers in each column, borrowing values from the column to the left when needed.

**Example:** 0.868 – 0.147.

Line up the decimal points, and then subtract just as if subtracting whole numbers.

$$\begin{array}{r} .868 \\ -\ .147 \\ \hline 0.721 \end{array}$$

**Example:** 7.068 – 4.159.

**Step 1:** Line up the decimal points.

$$\begin{array}{r} 7.068 \\ -\ 4.159 \\ \hline \end{array}$$

**Step 2:** Beginning to the right, subtract the numbers in the thousandths column. As 9 is greater than 8, we must borrow a 1 from the next column.
This gives us 18 – 9 = 9.

$$\begin{array}{r} {}^{1}8 \\ 7.06\not{8} \\ -\ 4.159 \\ \hline 9 \end{array}$$

**Step 3:** Subtract the numbers in the hundredths column. Since we borrowed 1, 6 must be reduced to 5.
We have 5 – 5 = 0.

$$\begin{array}{r} 5\,{}^{1}8 \\ 7.0\not{6}\not{8} \\ -\ 4.159 \\ \hline 09 \end{array}$$

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**Step 4:** Subtract the numbers in the tenths column. As 1 is greater than 0, we must borrow a 1 from the next column. This gives us 10 – 1 = 9.

$$\begin{array}{r} {}^{1}0\,5\,{}^{1}8 \\ 7.\not{0}\not{6}\not{8} \\ -\ 4.159 \\ \hline 909 \end{array}$$

**Step 5:** Subtract the numbers in the units, or ones, column. Since we borrowed 1, 7 must be reduced to 6. We have 6 – 4 = 2.

$$\begin{array}{r} 6\,{}^{1}0\,5\,{}^{1}8 \\ \not{7}.\not{0}\not{6}\not{8} \\ -\ 4.159 \\ \hline 2\ 909 \end{array}$$

**Step 6:** Bring down the decimal point.

$$\begin{array}{r} 6\,{}^{1}0\,5\,{}^{1}8 \\ \not{7}.\not{0}\not{6}\not{8} \\ -\ 4.159 \\ \hline 2.909 \end{array}$$

Solve 7.068 – 4.159 using your calculator.

[7] [•] [0] [6] [8] [–] [4] [•] [1] [5] [9] [=] 2.909



Use your calculator to check that the following answers are correct.

**6.428 – 5.907 = 0.521**

**16.625 – 3.75 = 12.875**

**0.98 – 0.076 = 0.904**

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 5

## MULTIPLICATION OF DECIMAL FRACTIONS

Multiplying decimals is the essentially the same as with whole numbers – the difference being in how you handle the decimal points. Here are the steps for multiplying decimal fractions.

1. To multiply decimals, perform the multiplications, ignoring the decimal points.
2. Once you complete the multiplication and have the product (or answer), add up the total number of decimal places in the original numbers.
3. Begin at the right side of the product, and count off the same number of places.

**Examples:** Multiply 0.75 × 0.5

$$\begin{array}{r} .75 \\ \times\ .5 \\ \hline .375 \end{array}$$

In the original numbers of the example, we have a total of three decimal places. There are two in 0.75 and one in 0.5, for a total of three. Beginning at the far right of the product, we count over three places and locate the decimal point in front of the 3. This gives us the answer of 0.375.

---

Multiply 0.875 × 0.25.

875 × 25 = 21875

There are 5 decimal places.

**Answer:** 0.21875

Solve 0.875 x 0.25 using your calculator.

**Examples:** Multiply $5.75 \times 3.125$, and round to the hundredths, or second, decimal place.

$575 \times 3125 = 1796875$

There are 5 decimal places. Beginning to the right of the 5 and counting over 5 places gives us an answer of 17.96875.

Given that the number to the right of the 6 (the hundredths position) is 8, we round 6 up to 7, and drop the other numbers.

**Answer:** 17.97

---

$15 \times 3.5$

Verify that the answer is 52.5.

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 6

## DIVISION OF DECIMAL FRACTIONS

Here is the process for dividing with decimals:

1. When dividing with decimals, the decimal point in the divisor is moved to the right until the divisor is a whole number.

2. The decimal point in the dividend is moved the same number of places to the right.

3. The place where the decimal point appears in the dividend is the same place where the decimal point appears in the quotient, or answer.

Note that when we move the decimal point to the right the same number of places in both the divisor and dividend, we do not change the value of the quotient, or answer.

**Example:** Divide 3.87 by 0.4 (3.87 ÷ 0.4).

There is 1 decimal place in the divisor. We move the decimal point 1 place to the right, giving us 4.

We also move the decimal point in the dividend 1 place to the right, giving us 38.7.

We now divide 38.7 by 4 (38.7 ÷ 4), making sure that we place the decimal point in the quotient in the same place as the decimal point in the dividend.

Follow the steps below to determine the answer to this problem.

**Step 1:** Divide 4 into 38. By inspection (based on the multiplication table), we determine that 4 will divide 9 times into the number 38. Multiply 9 × 4 = 36.

$$\begin{array}{r} 9\phantom{.7} \\ 4\,\overline{)\,38.7} \\ -\,36\phantom{.7} \\ \hline \end{array}$$

**Step 2:** Subtract 38 – 36 = 2. Bring down the 7, giving us 27.

```
      9
4 ) 38.700
  – 36
     27
```

**Step 3:** Divide 4 into 27, making sure to record the 6 to the right of the decimal point. Multiply 6 × 4 = 24.

```
      9.6
4 ) 38.700
  – 36
      27
    – 24
```

**Step 4:** Subtract 27 – 24 = 3. We have added several 0s following the 7. These do not change the value of the number and are used to continue to divide. Bringing down one of the 0s gives us 30.

```
      9.6
4 ) 38.700
  – 36
      27
    – 24
       30
```

**Step 5:** We continue to divide until we reach 0 (as in this problem) or until we have reached the **place value** or level of precision that we need. This means that in some problems you will need to round the answer.

```
      9.675
4 ) 38.700
  – 36
      27
    – 24
       30
     – 28
         20
       – 20
          0
```

Solve 3.87 ÷ 0.4 using your calculator.

[3] [·] [8] [7] [÷] [·] [4] [=] 9.675

Review and confirm the following examples using your calculator.

**Examples:** **0.2 ÷ 0.8**

Converting the divisor to a whole number gives us 2 ÷ 8. Performing the division gives us an answer of 0.25.

**0.625 ÷ 2.5**

Converting the divisor to a whole number gives us 6.25 ÷ 25. Performing the division gives us an answer of 0.25.

**6.052 ÷ 3.56**

Converting the divisor to a whole number gives us 605.2 ÷ 356. Performing the division gives us an answer of 1.7.

Here are some additional division problems. Be sure that you can verify the correct answers.

**6.5 ÷ 3.25 = 2**

**46.35 ÷ 45 = 1.03**

**51.012 ÷ 14.17 = 3.6**

**7.209 ÷ 0.9 = 8.01**

**80.64 ÷ 16.8 = 4.8**

**980.4 ÷ 2.15 = 456**

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.

# OBJECTIVE 7

## COMBINED OPERATIONS WITH DECIMAL FRACTIONS

When more than one of the four basic functions appears in a problem (addition, subtraction, multiplication, and division) a specific sequence must be followed in order to correctly solve the problem. This is referred to as the "mathematical order of operations."

### ORDERS OF OPERATIONS RULES

As a reminder, here are the rules for order of operations:

1. Simplify any expression inside a grouping symbol (e.g., parentheses and brackets).
2. Simplify expressions with exponents (exponents will be covered in the *Advanced Mathematics* course).
3. Carry out multiplication or division from *left to right*.
4. Simplify addition or subtraction from *left to right*.

**Note:** Scientific calculators are programmed to follow the rules for order of operations. Inexpensive, non-scientific calculators are not programmed to follow these rules and using them for these kinds of problems will result in incorrect answers.

UNIT 4

### COMBINED OPERATIONS

Review the following examples to see how to perform combined operations with decimal fractions.

**Example:** $4.7 + 3.6(8.1 \div 1.5) - 14 \div 2.5 =$

$4.7 + 3.6(5.4) - 14 \div 2.5 =$

$4.7 + 19.44 - 14 \div 2.5 =$

$4.7 + 19.44 - 5.6 = 18.54$

In the second line of the problem, we first divide the grouped expression $8.1 \div 1.5$ resulting in 5.4.

In the third line, we multiply $3.6 \times 5.4$ to get 19.44.

In the fourth line, we divide $14 \div 2.5$, and then subtract 5.6 from the sum of 4.7 and 19.44.

Solve the previous problem using your calculator.

4 · 7 + 3 · 6 ( 8 · 1 ÷ 1 · 5 ) − 1 4 ÷ 2 · 5 = 18.54

**Examples:** (4.2 ÷ 2.1) × 8.3 – 10 = **original problem**

| | |
|---|---|
| (4.2 ÷ 2.1) × 8.3 – 10 = | **original problem** |
| 2 × 8.3 – 10 | **divide 4.2 ÷ 2.1** |
| 16.6 – 10 | **multiply 2 × 8.3** |
| 6.6 = | **answer** |

| | |
|---|---|
| 256 – 3.2(4.8 ÷ 0.06) + 1 | **original problem** |
| 256 – 3.2 (80) + 1 | **perform division** |
| 256 – 256 + 1 | **perform multiplication** |
| 1 = | **answer (after subtraction and addition)** |

Here are some additional combined operation problems. Be sure that you can use your calculator to verify the correct answers.

**150 – 7.3 × 6.9 + 0.85 = 100.48**

**16 + 14.72 ÷ 4 – 19 = 0.68**

**[81 ÷ (6 × 1.5)] + 7.5 × 2 = 24**

**(1.8 × 2.6) ÷ (1.17 × 4) = 1**

**Note:** At this point you should be ready to complete the Assignment Sheet that will provide you with an opportunity to practice the skills learned in this section.